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論　文

On the Optimization of the Prototype Filter used in the Short Time DFT Hilbert Transformers

Short Time DFTヒルベルト変換にもちいられるプロトタイプフィルタの最適化

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ABSTRACT *Superior characteristics of the short time DFT Hilbert transformer is yielded through employing ideal proto-type filter of infinite frame number. From practical application of view, it becomes to be important to reduce frame number of this proto-type filter and the processing amount in Short Time DFT Hilbert transformers.*

Optimization of proto-type filters are discussed in this paper to realize small frame number filters equal to the infinite frame number filter in the functional facilities via both non-linear programming and functional analysis. The proto-type filters are optimized in the meanings of minimizing the ripples both on the pass- and elimination-bands and of getting the sharpness of the mainlobe. Optimized proto-type filters are verified with analyzing output signals through Hilbert transformers, which employ these filters, to yield both exactly unity in amplitude and half π radian in phase-shifting even if shorten frame number filters being employed.

1. INTRODUCTION

Such indispensable functions as nobel Hilbert transformers [1] and companders [2] are realized in the communication systems via employing instantaneous spectrum concept. This instantaneous spectrum is provided with the short time DFT (ab.in ST-DFT) which separates the frequency resolution from the time resolution, and makes accuracy of time resolution to extremity infinitesimal of single sampling duration.

A significant function in the ST-DFT is mainly based on the proto-type filters, which plays important role of the convolution for input data stream [3]. The proto-type filter defines preciseness of analyzing the input signals to be given by truncating infinite frame number Nyquist with N frame length into finite frame number $2m$.

Unfortunately, truncated Nyquist filters are suffered from Gibbs phenomenon to introduce such analyzing errors as excessive amplitude oscillations, interference to the adjacent frequency components, and etc. Optimizations are discussed in this paper in the meanings of Chebycheff with emphasis on reducing excessive oscillation and sharpening mainlobe to improve finite proto-type filters based on truncated Nyquist filters from two points of view; non-linear programming and functional analysis. Optimizing filters are verified through computer simulation how to improve the output signals of

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short time DFT Hilbert transformers.

2. DEFINITION OF THE PROTOTYPE FILTER

Necessary condition of the proto-type filter is deduced from specifications on the frequency domain,

$$H(e^{j\omega}) = \begin{cases} N, & -\frac{\pi}{N} \leqq \omega \leqq \frac{\pi}{N} \\ 0, & otherwise \end{cases} \tag{1}$$

Contol integral on eq.1 gives impulse response of the proto-type filter, that is,

$$\begin{aligned} h(n) &= \frac{1}{2\pi}\oint H(e^{j\omega})e^{j\omega n}d\omega = \frac{1}{2\pi}\int_{-\frac{\pi}{N}}^{\frac{\pi}{N}} Ne^{j\omega n}d\omega \\ &= \frac{\sin(\frac{n\pi}{N})}{\frac{n\pi}{N}}. \end{aligned} \tag{2}$$

Sufficient condition of the proto-type filter is given as follows [4],

$$h(p) = \begin{cases} 1, & if \quad p=0 \\ 0, & if \quad p=Nu, \quad u \text{ is non zero integer}. \end{cases} \tag{3}$$

The instantaneous spectrum $\Phi(n)$ is given by ST-DFT, that is,

$$\Phi(n) = \sum_{r=-\infty}^{\infty} x(r)h(n-r)W_N^{-rk}. \tag{4}$$

Here, n is sampling clock,

$x(r)$ is sampling data,

$h(*)$ is proto-type filter of ST-DFT, and

W_N^{-rk} is the same operator to the existing DFT.

On the frequency domain, Hilbert transformed spectrum $\widehat{\Phi}(n)$ is precisely given by merely exchanging the real part with imaginary and by going symmetrical structure with symmetric axis at index $k=N/2$ for the phisical existence of transformed signals [1]. Hilbert transformed signal $\widehat{y}(n)$ is, therefore, given by

$$\widehat{y}(n) = \frac{1}{N}\sum_{k=0}^{N-1} \widehat{\phi}_k(n)W_N^{nk}. \tag{5}$$

Here, $\widehat{\phi}_k(n)$ is frequency component of $\widehat{\Phi}(n)$ at frequency index k.

If and only if $h(*)$ holds on eq.3, the output signal $y(n)$ is exactly reproduced from the instantaneous spectrum $\Phi(n)$.

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1}\sum_{r=-\infty}^{\infty} x(r)h(n-r)W_N^{-rk}W_N^{nk}. \tag{6}$$

Since output $y(n)$ is given by linear operations on finite operand, it holds on exchanging the summation order for k with r,

$$\begin{aligned} y(n) &= \sum_{r=-\infty}^{\infty}\frac{1}{N}\sum_{k=0}^{N-1} x(r)h(n-r)W_N^{(n-r)k} \\ &= \begin{cases} \sum_{r=-\infty}^{\infty} x(r)h(n-r) = x(n) \\ 0, \qquad else. \end{cases} \end{aligned} \tag{7}$$

If transformed spectrum $\widehat{\Phi}(n)$ is substituted into eq.6 instead of $\widehat{\Phi}(n)$, eq.6 will also express Hilbert transformed signal $\widehat{y}(n)$.

Fig.1 shows frequency response of truncated Nyquist as proto-type filter where frame number *2m* is varied from 0 to infinite number. As clearly shown in fig.1, infinite frame number Nyquist behaves as ideal proto-type filter with significant victim of paying infinite processing amount during convolution of input signals. When the frame number becomes to be smaller, amplitude peak values both of main and side lobes become to be greater, and the sharpness of cut-off becomes to vague to show Gibbs phenomenon in adjacent channels.

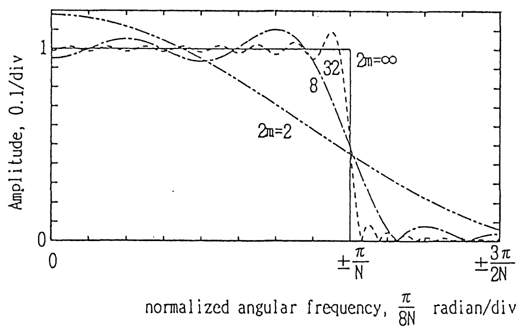

Fig. 1 Amplitude frequency responses of the truncated Nyquist vs. frame number *2m*

3. OPTIMIZATION OF THE PROTOTYPE FILTER

3.1 Steepest Gradient Method

Truncated Nyquist is optimized via the steepest gradient method under the restriction condition of eq.3, where $h(*)$ is kept to be null at every N point. Initial value vector X_0 are chosen to those of truncated Nyquist as follows,

$$X_0 = \{h(-mN),\ h(-mN+1),\ \cdots,\ h(mN)\}^T. \tag{8}$$

Iterations are carried over for current vector X_i to give new vector X_{i+1} by following successive equation,

$$X_{i+1} = X_i - \delta grad X_i. \tag{9}$$

Here, $gradX_i$ is gradient vector for vector X_i .

$$gradX_i = m(p)\frac{\delta J(X_i)}{\delta X_i}. \tag{10}$$

Where,

$$J(X_i) = \max_{\Omega \in S} |H_d(\Omega) - H(\Omega)|, \quad S = \{\Omega \mid \omega_L \leqq \Omega \leqq \omega_U\}. \tag{11}$$

X_i is i-th iteration coefficient vector, $J(X_i)$ is evaluation value defined by maximum value of ripples on the pass-band. $H_d(\Omega)$ is the desired frequency response, $H(\Omega)$ is frequency response for the coefficient vector X_i , ω_L and ω_U are lower and upper normalized angular frequencies, and δ is a stepping quantity, and $m(p)$ is weighting function which ensures the continuously along to the coefficient values,

$$m(p) = \frac{1}{2}\left\{1 - \cos\frac{2\pi p}{N}\right\}, \quad -mN \leqq p \leqq mN. \tag{12}$$

Weighting function $m(p)$ is significant to keep the continuously over coefficient values. Unless $m(p)$ is employed, discontinuity over coefficient $h(*)$ occurs at every N point owing to the restriction of eq.3. Discontinuity over coefficients of proto-type filter $h(*)$ yields echos on the time domain response to degrade input signals during ST-DFT convolution. The values of gradient components are however chosen not to vanish around every N point according to the restriction of eq.3. After many iterations, the value of coefficients goes to large in absolute around every N point, while value at just every N point is kept to be null under eq.3 restrictions. Consequentry, discontinuity appeares around at every N point.

Fig.2 shows amplitude frequency response of 8 frame ($2m$=8) proto-type filter optimized with steepest gradient method to hold continuity on the coefficient values. As shown in fig.2, the amplitude ripples on the pass-band are improved to be smaller than 0.300 dB and the elimination is greater than 25.135 dB, while the amplitude ripple and elimination of the original truncated Nyquist of 8 frame are 0.858 and 22.125 dB.

Fig.3 shows the maximum amplitude ripples

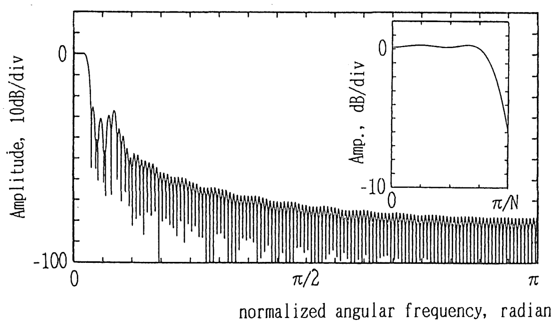

Fig. 2 Amplitude frequency responses of optimized filter by the steepest gradient method, $2m$=8

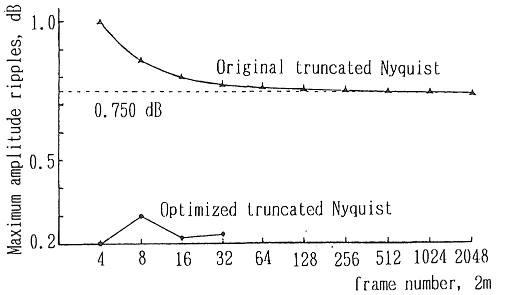

Fig. 3 Maximum ripples on the pass-band both of the original and optimized truncated Nyquist vs. frame number $2m$

both of the optimized and original truncated Nyquist on the pass-band as frame number $2m$ being taken to be parameter. The values of original truncated Nyquist becomes to saturate close to 0.75 dB as frame number $2m$ goes over 32. Optimization given by eqs. from 8 to 11 improves the maximum amplitude below 0.300 dB even if frame number $2m$ is taken to be less than 32.

3.2 Functional Analysis Approach

Let's consider what effect will be introduced by smoothing truncated Nyquist over all on the $2m$ frames as follows,

$$h_m(p) = h_N(p)m(p). \qquad (13)$$

Here, $h_N(p)$ is the truncated Nyquist with $2m$ frames, $m(p)$ is, for example, smoothing function of Hanning.

Attention must pay on the frame length of $m(p)$ being taken as $2mN$ instead of N.

Fig.4 shows clearly the effect induced from modifing 8 frame truncated Nyquist by smoothing of Hanning. Solid line shows amplitude frequency response of smoothed Nyquist by Hanning $h_{Nn}(p)$. Dotted line shows amplitude frequency response of non-smoothed Nyquist. That is, it concentrations mainlobe to the origin of frequency domain what truncated Nyquist is smoothed to vanish its value at both tail-ends on the$2m$ frames.

Kaiser is successfully employed as smoothing function $m(p)$ to get mainlobe energy more large. As well known, Kaiser gets largest energy in mainlobe for a given arbitrary value of sidelobes. Smoothing function $m(p)$ is given by eq.14,

$$w(p) = \frac{I_0\left\{\beta\sqrt{1-(p/mN)^2}\right\}}{I_0(\beta)}, -mN \leqq p \leqq mN. \qquad (14)$$

Here, $I_0(*)$ is the modified 0th order Bessel of the first kind, β is arbitrary value to adjust width or energy of the mainlobe.

Hereafter, truncated Nyquist smoothed with Kaiser is called by "Nyquist-Kaiser". Fig.5 shows that minimum attenuation of the Nyquist-Kaiser of β=7.865, on the elimination-band is more than 79.2 dB, and that width of the mainlobe is 0.085 radian almost equal to that of infinite frame number Nyquist given by eq.2, and the maximum ripple on the pass-band is less than 0.001dB. Consequently, the truncated Nyquist smoothed by Kaiser improves characteristics of proto-type filter almost equal to ideal filter without any victim of paying excessive computing power during ST-DFT. Value of the

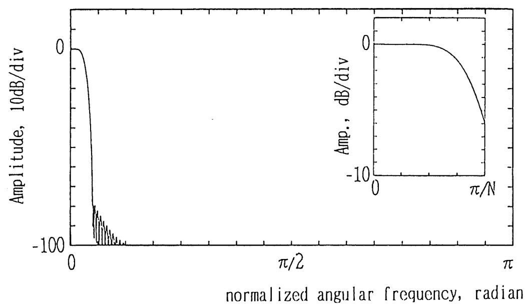

Fig.5 Amplitude frequency response of Nyquist-Kaiser, $2m$=8, β=7.865

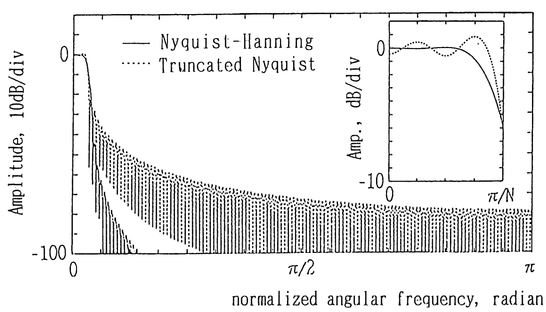

Fig.4 Amplitude frequency response of both Nyquist-Hanning and truncated Nyquist, $2m$=8

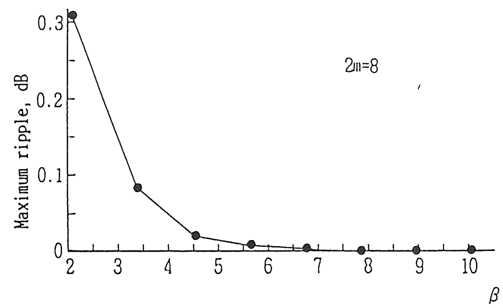

Fig.6 Maximum ripple on the pass-band vs. arbitrary value β

maximum ripple on the pass-band vanishes from 0.3dB to zero as shown in fig.6, where the arbitrary β increases from 2.12 to 10.06. The maximum ripple of Nyquist-Kaiser is observed to vanish beyond β=4.534.

Fig.7 shows the minimum attenuation on the elimination-band of Nyquist-Kaiser where β being taken as a parameter. The minimum attenuation monotonically increases in the meaning of decibel from 31.1 to more than 99.4 dB, when arbitrary β goes from 2.12 to 10.06. Such smoothing function as Kaiser employed in the truncated Nyquist is shown as discussed aboves to reduce maximum ripple on the pass-band and to increase attenuation on the elimination-band almost equal those of infinite frame number Nyquist. Nyquist-Kaiser of finite frame number is consequently recognized as an optimum proto-type filter in the meaning both of minimum ripple and maximum attenuation under restriction of given frame number.

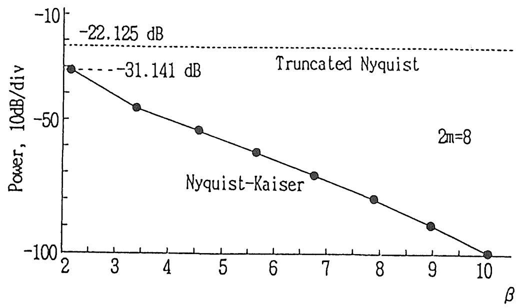

Fig. 7 Characteristics of minimum suppression on eliminating domain for β

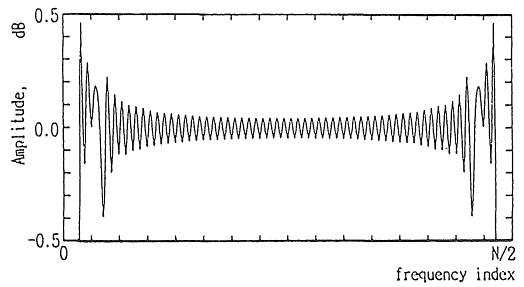

Fig. 8 Amplitude frequency response of the short time DFT Hilbert transformer which employ 8 frame Nyquist optimized by steepest gradient, N=32

4. CHARACTERISTICS OF THE HILBERT TRANSFORMERS

Two type of optimized prototype filters are verified with employing in the ST-DFT Hilbert transformers. Fig. 8 shows amplitude frequency response of the ST-DFT Hilbert transformer which employs the 8 frame proto-type filter optimized by the steepest gradient method. The maximum amplitude ripple on the pass-band is examined to be 0.465 dB with improvement of more than 0.37 dB from original truncated Nyquist. The maximum ripple of amplitude error on the pass-band is shown in fig.9 to be less than 0.5 dB of that of infinite frame number Nyquist over the range of frame number beyond $2m$=8.

Fig.10 shows amplitude frequency response of the ST-DFT Hilbert transformer which employs the same 8 frame proto-type filter smoothed by Kaiser of β=7.865. The maximum amplitude ripple on the pass-band is observed to be less than 0.0007 dB to hold unity on almost all pass-band. The maximum ripple of employing Nyquist-Kaiser is shown in fig.11 to vanish in dB unit as arbitrary β beyond 5. Attention must be payed on that the maximum ripple of the infinite frame number Nyquist is more than 0.75 dB on the pass-band. On the other hand, Nyquist-Kaiser reduces the maximum ripple on the pass-band below 0.001 dB only using 8 frame if β is set to be 10.056.

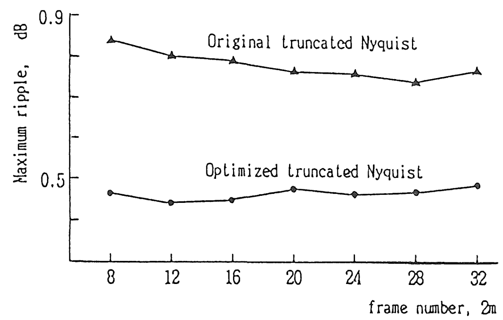

Fig. 9 Maximum ripple of the short time DFT Hilbert transformer which employs optimized proto-type filter via steepest gradient method

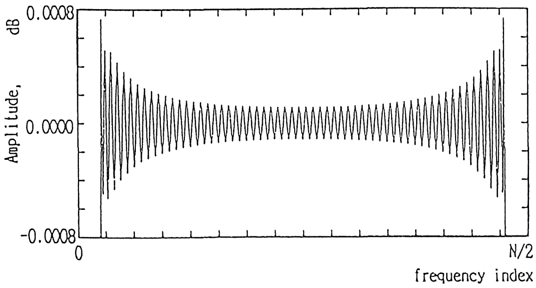

Fig. 10 Amplitude frequency response of the short time DFT Hilbert transformer which employ 8 frame Nyquist-Kaiser of $\beta=7.865$, $N=32$

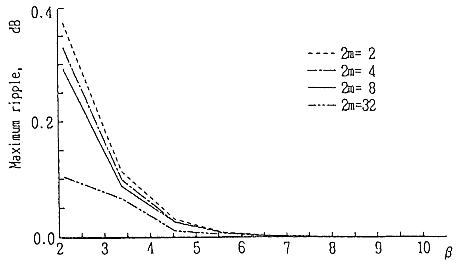

Fig. 11 Maximum ripple on the pass-band of the short time DFT Hilbert transformer which employs Nyquist-Kaiser vs. β

5. CONCLUSION

The optimizations of proto-type filter were discussed in this paper for reducing significant function in the ST-DFT with emphasis on both reducing ripples of pass- or elimination-band and sharpening mainlobe to improve small frame number filters through the steepest gradient method and functional analysis. These optimized proto-type filters ensures that the ST-DFT Hilbert transformers are realized to be almost free from any distortions for keeping preciseness on instantaneous spectrum analysis. Further studies will be optimized to wide the pass-band of proto-type filter nearly equal to the same frame numbers truncated Nyquist.

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