

# A Proposal of Short Time DFT Syllabic Compressor and Its Configurations

## Short Time DFTコンパンダの提案とその構成

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**ABSTRACT** The syllabic compander reduces fading noise to improve speech quality over poor radio channels. However, conventional syllabic companders are suffered from such distortions as harmonics and intermodulations owing to employing AM demodulator to estimate signal envelope.

Short time DFT syllabic compander is newly proposed here with emphasis on employing concept of instantaneous spectrum to detect envelopes instead of approximation of AM demodulation in realizing the envelope detectors. The instantaneous spectrum analysis, which separates time resolution from frequency resolution, puts the short time DFT compander on the stage of developing exact companders for the radio communication systems.

The short time DFT compander is discussed in this paper with emphasis on how to operate on the frequency domain and how to get efficient processing against tremendously great amount during analyzing instantaneous spectrum. Multi-rate sampling is also successfully employed to speed up analysis in the short time DFT compander without almost any degradation.

### 1. INTRODUCTION

Fig.1 shows how syllabic companders improve the speech quality on the poor noisy channels. Speech quality is as well known degraded over poor radio channels with such noise as fading, and thermal, etc. Non linear operation of ex-

panding input signals is able to recover the degraded speech quality by twice in the meaning of decibel as shown in level diagram of fig.1.

Now, consider the conventional syllabic compander shown in fig.2 how to improve speech quality of degraded signals. Let  $x(t)$  be input signals,  $y(t)$  be output signals of conventional syllabic compressor, and  $z(t)$  be output signals of conventional expander. Where expander is directly connected to compressor, output  $z(t)$  is given as follows.

$$z(t) = y(t) \times E\{y(t)\}. \quad (1)$$

$$y(t) = x(t) \div E\{y(t)\}. \quad (2)$$

Here,  $E\{*\}$  means envelope of the signal  $*$ .

It is easy to understand that relation

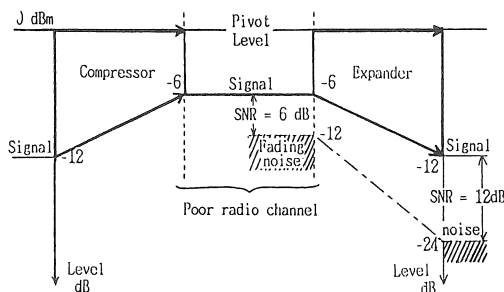


Fig. 1 Level diagram over poor radio channels.

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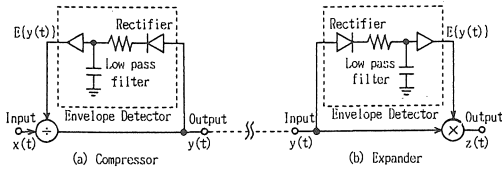


Fig. 2 Configurations of conventional companders.

$E\{E\{*\}\} = E\{*\}$  holds on the envelope detection, when envelope detector is remembered to consist of two major parts, rectifier and low pass filter as shown in fig.2.

Taking envelopes on the both hands of eqs.1 and 2, it gives significant equations as follows,

$$E\{z(t)\} = E\{y(t)\}^2, \tag{3}$$

$$E\{y(t)\}^2 = E\{x(t)\} \rightarrow E\{y(t)\} = E\{x(t)\}^{\frac{1}{2}}. \tag{4}$$

Eqs.3 and 4 show clearly the function of expanding 1 to 2 in decibel meaning or of compressing 2 to 1.

Since both signals and noise of input signals for the expander are expanded by 1 to 2 in decibel meanings, the speech quality is shown to be recovered by twice in SNR meanings so long as SNR is greater than 0dB. Both envelope detector and feed-back loop are literally recognized to play important role in conventional companders.

Fig.3 shows the feed-forward structure to exclude feed-back loop from the compressor to be shown in ref.1. Where root-operator(rooter) is employed, feed-back loop is excluded as shown in fig.3 from the conventional syllabic compressor to yield the same functions to the feed-back compressor shown in fig.2(a). That is, the output  $y(t)$  is given as,

$$y(t) = x(t)/E\{x(t)\}^{\frac{1}{2}}. \tag{5}$$

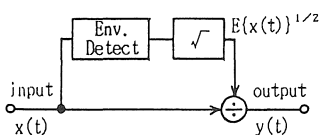


Fig. 3 Feed-forward structure which excludes feed-back loop from the conventional syllabic compressor shown in fig.2(a).

Similar to eq.4, envelope of eq.5 is given as follows,

$$E\{y(t)\} = E\{x(t)\}^{\frac{1}{2}}. \tag{6}$$

Eq.6 shows that the structure of feed-back loop of conventional syllabic companders does not play inevitable roles in compressing signals. Moreover, eq.6 suggests the existence of new type compressor, which excludes feed-back loops and even envelope detector.

## 2. PRINCIPLE OF THE SHORT TIME DFT COMPANDER

Now, let's consider all sampled signals to be described as follows,

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} \phi_k(n) e^{j\frac{2\pi}{N}nk}. \tag{7}$$

Here,  $\phi_k(n)$  is frequency component of instantaneous spectrum  $\Phi(n)$  at sampling clock  $n$ .

Every instantaneous spectrum component  $\phi_k(n)$  is given by short time DFT as follows[2],

$$\phi_k(n) = \sum_{r=-\infty}^{\infty} x(r)h(n-r)e^{-j\frac{2\pi}{N}kr}. \tag{8}$$

Where,  $x(r)$  is sampled data at clock  $r$ ,  $N$  is frame length which defines frequency resolution in the same meaning of conventional DFT,  $k$  is frequency index,  $0 \leq k < N$ .

On the frequency domain, every component  $\phi_k(n)$  is illustrated in fig.4. Compressing  $\phi_k(n)$  are already shown to perform by dividing  $\phi_k(n)$

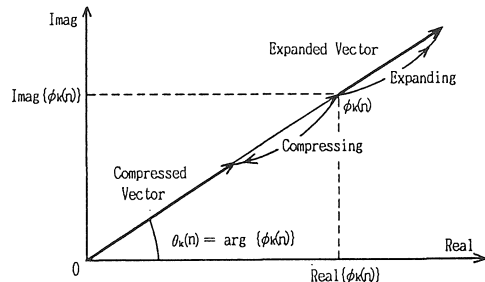


Fig. 4 Comanding the instantaneous spectrum component on the frequency domain.

with  $|\phi_k(n)|^{\frac{1}{\alpha}}$  along to the vector  $\phi_k(n)$  [3].

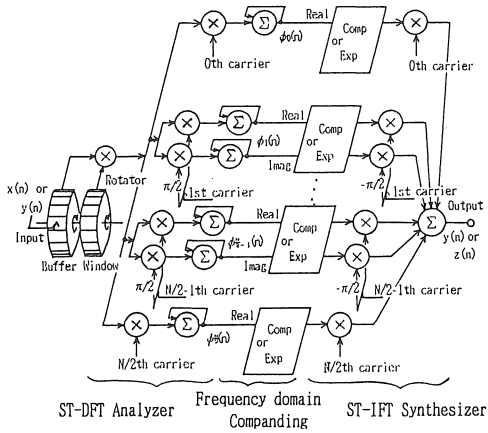
The compressed signals  $y(n)$  are consequently given as follows,

$$y(n) = \frac{1}{N} \left\{ |\phi_0(n)|^{1-\alpha} + \sum_{k=1}^{N-1} \frac{\phi_k(n)}{|\phi_k(n)|^\alpha} e^{j\frac{2\pi}{N}nk} \right\}. \quad (9)$$

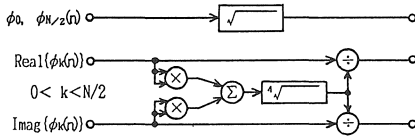
Where compressing rate is required to be 2 in decibel meanings to adopt to the conventional companding systems,  $\alpha$  becomes  $\frac{1}{2}$ . If arbitrary compressing rate is required,  $\alpha$  is sufficient to be set to the reciprocal number of required value.

The expanding is also shown in fig.4 as instantaneous spectrum expansion as follows,

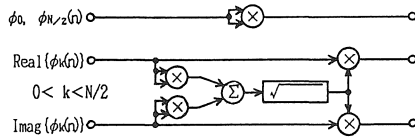
$$z(n) = \frac{1}{N} \left\{ \phi_0(n)^\beta + \sum_{k=1}^{N-1} |\phi_k(n)|^{\beta-1} \phi_k(n) e^{j\frac{2\pi}{N}nk} \right\}. \quad (10)$$



(a) Circuitry configuration of ST-DFT compander



(b) Details of the compressor



(c) Details of the expander

Fig. 5 Circuitry configuration of ST - DFT compander, detailed of compressor (b), expander (c).

Here,  $\beta$  is expanding rate in dB meanings, and  $\phi_k(n)$  is defined as the instantaneous spectrum of the input signals  $y(n)$  to the expander.

As discussed above, both compressed and expanded signals are themselves denoted by the same formula of instantaneous spectrum expansion. It is easy to understand that the short time DFT compander ensures to be free from any distortion in companding processing.

The circuitry configuration becomes modulo-structure along to frequency index as shown in fig.5. Owing to employing short time DFT, the short time DFT compander are released from approximation in detecting the envelopes.

### 3. PROCESSING OF THE SHORT TIME DFT COMPANDER

#### 3.1 Based on FFT Structure

As discussed in the previous session, short time DFT compander is theoretically free from any distortion owing to employing instantaneous spectrum concept. Unfortunately, it is suffered from great deal of computing to get the instantaneous spectrum. Further investigation is keenly studied on reducing the computational power in the short time DFT companders.

At first, modulo structure is deduced from the short time DFT by setting variables  $r = n + s$ , and  $s = lN + m$ ,

$$\begin{aligned} \phi_k(n) &= \sum_{n+s=-\infty}^{\infty} x(n+s)h(-s)W_N^{-(n+s)k} \\ &= W_N^{-nk} \sum_{n+lN+m=-\infty}^{\infty} x(n+lN+m)h(-lN-m)W_N^{-(lN+m)k} \\ &= W_N^{-nk} \sum_{m=0}^{N-1} \sum_{l=-\infty}^{\infty} x(n+lN+m)h(-lN-m)W_N^{-mk}. \end{aligned}$$

That is, eq.8 is modified to be described by FFT formulation as follows,

$$\phi_k(n) = \sum_{m=0}^{N-1} x_m(n)W_N^{-mk}, \quad W_N^{-mk} = e^{-j\frac{2\pi}{N}mk}. \quad (11)$$

Here,

$$x_m(n) = \sum_{l=-\infty}^{\infty} x(n + lN + [m - n]_N)h(-lN - [m - n]_N),$$

$$[M]_N = M \text{ mod } N. \quad (12)$$

Eq.11 reduces the computing amount to  $O(N \log N)$ . Since conventional DFT requires computing amount of  $O(N^2)$  and FFT requires  $O(N \log N)$ , the instantaneous spectrum  $\phi_k(n)$  is given after computing of  $O(N^2)$  based on DFT of eq.8, and given after computing of  $O(N \log N)$  based on FFT structure of eq.11.

### 3.2 Based on Frequency Domain Interpolation

As shown in the definition of short time DFT, the instantaneous spectrum are deduced from  $2m$  times  $N$  sampled data, within single frame of conventional DFT or FFT.

Where only somewhat degradation is allowed to fast processing from practical application of view,  $\tilde{\phi}_k(n)$  may be exactly reproduced from only thinned out  $\phi_k(rR)$  at every  $R$  sampling clock as follows,

$$\tilde{\phi}_k(n) = \sum_{r=L^-}^{L^+} f(n - rR)\phi_k(rR). \quad (13)$$

Here,  $f(n - rR)$  is, for example, Lagrange interpolation of  $2Q$  frame given by,

$$f(n - rR) = \frac{(-1)^{r+Q}}{(Q-1+r)!(Q-r)!\left(\frac{n}{R} - r\right)} \prod_{i=1}^Q \left(\frac{n}{R} + Q - i\right), \quad (14)$$

$$L^- = \left\lfloor \frac{n}{R} \right\rfloor - Q, \quad L^+ = \left\lceil \frac{n}{R} \right\rceil + Q - 1. \quad (15)$$

$[*]$  represents the largest integer contained  $*$ .

The output signals  $\tilde{y}(n)$  are consequently synthesized through eq.16, where frequency domain interpolation are employed.

$$\tilde{y}(n) = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{\phi}_k(n) W_N^{nk}, \quad W_N^{nk} = e^{j\frac{2\pi}{N}nk}. \quad (16)$$

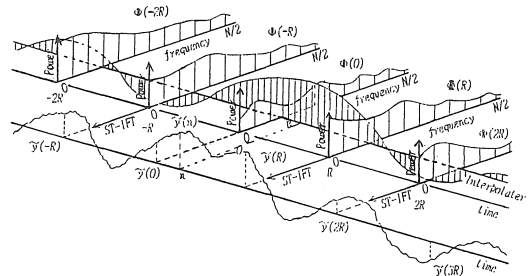


Fig. 6 Fast processing diagram in the ST-DFT compander based on frequency domain interpolation.

Eq.16 gives such a processing structure as shown in fig.6, which reduces the processing amount to  $(2mN + 2N \log N)/R$ . Where  $R$  becomes close to  $N$  with troublesome appearance of Gibb's phenomena, the processing is mostly speeded up by amount  $N$  times.  $R$  is recommended to be less than  $N$  of the frame length.

### 3.3 Based on Time Domain Interpolation

Substituting eq.13 into eq.16, the output signals  $\tilde{y}(n)$  are given as,

$$\tilde{y}(n) = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{r=L^-}^{L^+} f(n - rR)\phi_k(rR)W_N^{nk}. \quad (17)$$

Since all of the operations in eq.17 are linear on the finite operand, eq.17 holds on exchanging the order of summation for  $k$  with  $r$ .

$$\tilde{y}(n) = \sum_{r=L^-}^{L^+} f(n - rR)s_r(n) \quad (18)$$

Here, every  $s_r(n)$  is time domain signals, synthesized via short time IFT from  $\phi_k(rR)$ , that is,

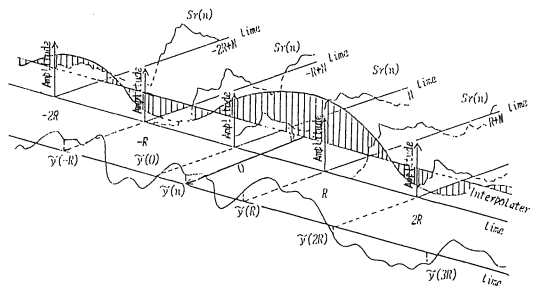


Fig. 7 Fast processing diagram in the ST-DFT compander based on time domain interpolation.

$$s_r(n) = \frac{1}{N} \sum_{k=0}^{N-1} \phi_k(rR) W_N^{nk}, \quad L^- \leq r \leq L^+. \quad (19)$$

Eq.18 shows clearly that output signals  $\tilde{y}(n)$  are reproduced with interpolating the sub-time domain signals  $s_r(n)$  as shown in fig.7.

### 3.4 Computing Amount Reduction via Interpolations

Two sophisticated fast algorithms are discussed in the aboves based on frequency domain and on time domain interpolation. Comparing with each other, processing amount  $V$  is considered in the unit of a real number product sum. The short time DFT companders are estimated to require the following computing amount at every sampling clock,

$$V_F = \frac{2mN + 2N \log N}{R} + c_e \frac{(N/2 + 1)}{R} + \frac{4Q(N/2 + 1)(R - 1)}{R} + 2\left(\frac{N}{2} + 1\right). \quad (20)$$

$$V_T = \frac{2mN + 2N \log N}{R} + c_e \frac{(N/2 + 1)}{R} + \frac{2Q(R - 1)}{R} + \frac{2}{R} \left(\frac{N}{2} + 1\right). \quad (21)$$

Here,

$$c_e = \begin{cases} 38, & \text{if } V \text{ is concerned with} \\ & \text{12bit resolution compressor.} \\ 10, & \text{if } V \text{ is concerned with} \\ & \text{12bit resolution expander.} \end{cases} \quad (22)$$

Suffix  $F$  or  $T$  of  $V$  means computing amount of the fast processing algorithm based on frequency or time domain, respectively.

The first term of eq.20 or 21 on right hand means the processing amount of the short time analyzer, the third means the amount of interpolation, and the last means that of short time synthesizers. The second term of eq.20 means value of processing amount in the short time DFT compressor or expander based on frequency

domain fast algorithm, and the second of eq.21 means that of processing amount in the short time DFT compressor or expander based on time domain fast algorithms.

Fig.8 shows computing amount as  $R$  being taken as parameter, here frame number of decimation filter  $h(*)$   $2m$  is set to be 8, frame number  $2Q$  of interpolation  $f(*)$  is 8, and both of frame length  $N$  are 32. The figure is featured in monotonic decrease both of  $V_F$  and  $V_T$  all over the interpolation duration  $R$  from 1 up to  $N$ .

In the case of  $R=N$ , where processing error becomes worst, total computing amount of the fast short time DFT compressor or expander is reduced below 26.7% or 41.1% via interpolation on the frequency domain in comparison of without interpolation. Furthermore, via time-domain interpolation, it is catastrophically reduced below 3.7% or 4.1% in the short time DFT compressor or expander.

## 4.CHARACTERISTICS OF THE FAST SHORT TIME DFT COMPANDER

The short time DFT compressor and expander are substantiated to be ideal in companding signals through computer simulations on CRAY

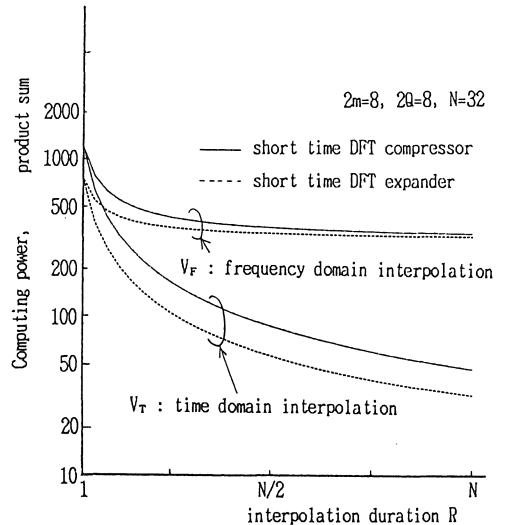


Fig.8 Computing power of the ST-DFT compander vs. interpolation duration  $R$ .

X-MP/14se at AIT to avoid round-off errors under CCITT G.162 specifications[5].

### 4.1 Operating Characteristics

Fig.9 shows three operating characteristics. The first is observed at the output of the short time DFT compressor, when 800 Hz tonal input signals are adopted at level from -80 to 0dBm. The second is observed at the output of the expander directly cascaded to the compressor for the same 800 Hz input. The last is observed at the output of the expander, which operates separately, when 800 Hz tonal input signals are adopted at level from -40 to 0dBm. Where 0 dBm is chosen as the pivot level, and sampling frequency is 8 kHz.

All are observed under the condition of decimation frame number  $2m=8$ , frame length  $N=32$ , interpolation frame number  $2Q=8$ , and interpolation duration  $R=N$ .

As shown in fig.9, the output levels are exactly on straight-line without any displacements. That is, both of the short time DFT compressor and expander operate so precisely as novel compressors with almost equivalent level of quantization error without employing envelope detector.

### 4.2 Harmonic Distortion

Harmonic distortion, measured with 800Hz

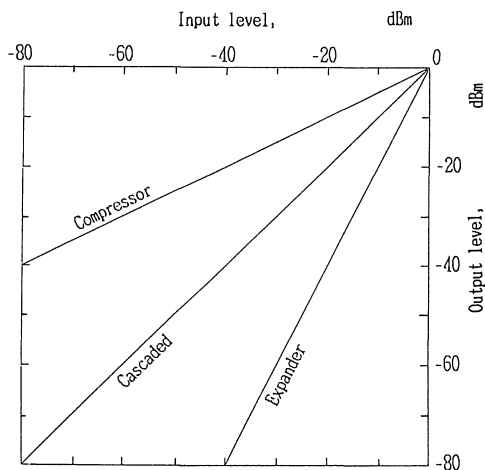


Fig. 9 Operating characteristics of the ST - DFT compressor, input signal is tonal 800 Hz.

0dBm tonal signals, is recommended to be below 4%, i.e. -14dB. Fig.10(a) shows clearly that the maximum distortion in the power spectrum appears at 2.4kHz as the third harmonic below -23.3dBm, and the second value at 4.0kHz as the fifth harmonic below -26.3dBm. The harmonic distortion of the fast ST-DFT compressor are observed to be below -21.5dB with more than 7.5dB margin to the criterion, when interpolation duration  $R$  is set to be 5.

Where  $R$  is set to be up to 10 to make the processing speed faster, the excessive harmonic distortions appear as shown in fig.10(b) at 0Hz below -21.5dBm, at 1.6 kHz as the second harmonic below -23.26dBm, and at 3.2 kHz as the fourth harmonic below -31.27dBm, while the third and fifth harmonics decrease their value to -29.04

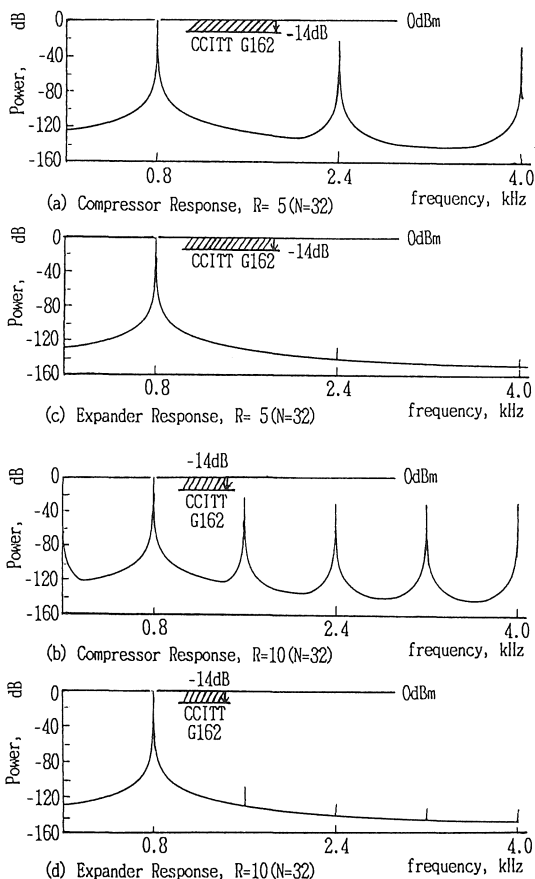


Fig. 10 Power spectra of the fast ST - DFT compressor, (a) of the compressor of  $R=5$ , (b) of  $R=10$ , and (c) of the expander of  $R=5$ , (d)  $R=10$ .

and -31.91dBm. These excessive harmonics suggest processing error which are mainly introduced from interpolation. The harmonic distortion of the fast ST - DFT compressor are consequently seemed to be below -18.4dB with 4.4dB margin.

Fig.10(c) shows the frequency responses of the fast ST - DFT expander which operates separately. In the figure, the maximum distortion appears at 2.4 kHz as the third harmonic below -128.5dBm, and the second distortion appears at 4.0 kHz as the fifth below -138.6dBm. The harmonic distortion of the fast ST - DFT expander of interpolation duration  $R=5$  is observed to be below -128.5dB with more than 114.5dB margin to the -14dB criterion.

Where  $R=10$ , the excessive distortions are observed at 1.6 kHz as the second harmonic below -107.4dBm, at 3.2 kHz as the fourth below -132.3dBm, while both the third and fifth harmonic almost maintain their values. The harmonic distortion of the expander of  $R=10$  is thereby seemed to be below -107.4dB with more than 93.4dB margin to CCITT recommendation.

### 4.3 Intermodulation Tests

The intermodulation signal level, which seems to be adequate for signalling system No.5, is also recommended to be below -26dB at frequency  $2f_1 - f_2 (=f_L)$  and  $2f_2 - f_1 (=f_U)$  for compressor or expander which operates individually. Here, input signal  $f_1$  and  $f_2$  are defined to be 900 Hz and 1020 Hz both of -5dBm or -15dBm.

However, it exceeds the limit of intermodulation tests as  $N=32$ , because both 900 and 1020 Hz signals belong to the same 4th sub-channel of ST - DFT so long as  $N=32$ .

On the other hand, if  $N=64$  and the sampling rate is remained to 8 kHz, the bandwidth of the ST - DFT reduces to 125 Hz, and each signal at 900 and 1020 Hz becomes to distinguished sub-channel.

It is shown in figs.11(a) and (b) for the specified signals that the intermodulation in the fast

ST - DFT compressor of  $R=5$  is at level of -37.08dB on the frequency  $f_L$  and -31.39dB on the frequency  $f_U$  with more than 11.08dB and 5.39dB margin to the CCITT specifications.

It is also shown in figs.11(c) and (d) for the

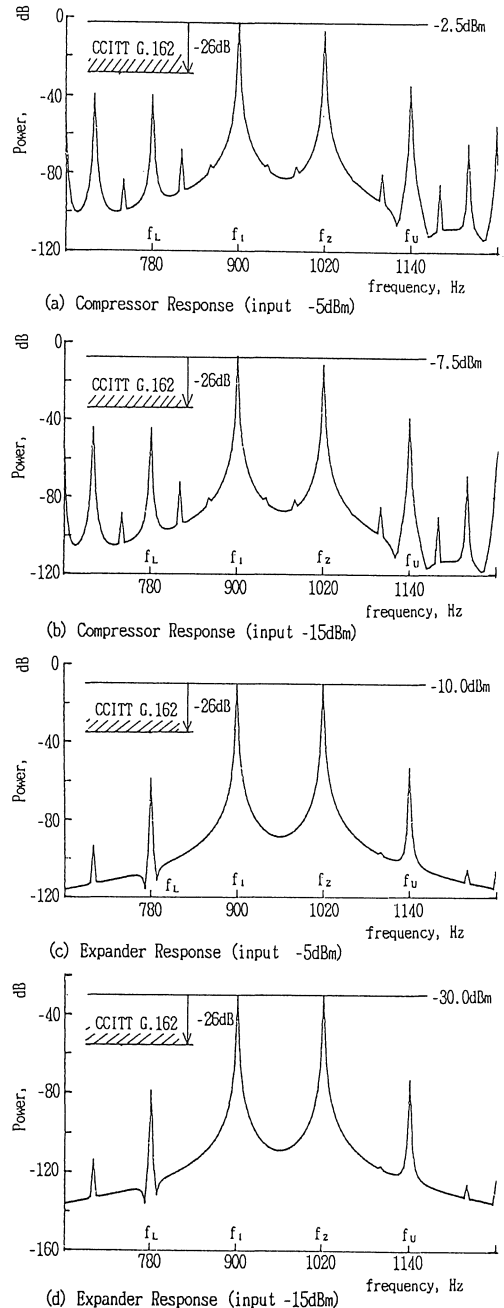


Fig. 11 Power spectra of the fast ST - DFT compander, (a) of the compressor for -5 dBm input, (b) for -15 dBm input, and (c) of the expander for -5 dBm input, (d) for -15 dBm input.

same specified signals that the intermodulation of the fast ST-DFT expander of  $R=5$  is at level of  $-48.77\text{dB}$  on the frequency  $f_L$  and  $-43.04\text{dB}$  on the frequency  $f_U$  with more than  $22.77\text{dB}$  and  $17.04\text{dB}$  margin to the criterion.

## 5. CONCLUSION

A novel compander was discussed on the concept of instantaneous spectrum with emphasis of fast algorithm, through its circuitry configuration, operating characteristics, harmonic distortion, and intermodulation. Short time DFT is successfully discussed to realize novel processing of companding on the frequency domain. Multi-rate sampling is also efficiently employed to get fast algorithms in the short time DFT expanders. The fast short time DFT expanders are shown to be almost free from any distortions within reasonable interpolation duration.

Further studies on optimizing the decimation filter of the short time DFT will make the duration more long to speed up short time DFT com-

panders.

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(受理 平成4年3月20日)