A Study on Optimization of the Decimation Filter Used in Short Time DFT Hilbert Transformers

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Short Time DFT Hilbert 変換における最適化 Decimation フィルタ合成の一考察

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The decimation filter plays significant roles in the short time DFT Hilbert Transformers. While the longer windows introduces the sharper cut-off characteristics, on the other hand, such longer windows introduce the longer delay in the response. Therefore, the decimation filters with sharp cut-off response have been keenly investigated to reduce the window length.

This paper shows a decimation filter optimized by the steepest gradient method in the mean of minimizing maximum ripple on the passband.

1. Introduction

The decimation filters play important roles in the short time DFT (ab. in ST-DFT) on the practical stage of digital signal processing related to the instantaneous spectrum analysis(1). The longer window promises the sharper cut-off characteristics in the frequency response in contrast with the longer delay in the signal processing. In addition to the processing delay, the long frame window yields complexity in implementations of circuit.

On the other hand, the shorter window promises the quick response in signal processing in contrast with losing the sharpness of cut-off response on the frequency domain(2). However, the Chebyshev characteristics will release the frequency response from losing sharpness. That is, the well-controlled scattering ripples on the passband of the window introduce existence of an optimization for the decimation filter.

2. ST-DFT Hilbert Transformers

The ST-DFT gives the instantaneous spectrum, $\hat{\Phi}(n)$, at sampling time n as follows(1),

$$\hat{\Phi}(n) = [\hat{\phi}_0(n)\hat{\phi}_1(n)\cdots\hat{\phi}_{N-1}(n)]^T \quad (1)$$

Here, the frequency component $\hat{\phi}_k(n)$ of $\hat{\Phi}(n)$ is given as follows,

$$\hat{\phi}_k(n) = \sum_{r=-\infty}^{\infty} x(r)w(n-r)\hat{W}_N^{-kr},$$

integer k is $0 \leqq k < N$. (2)

Here, $x(r)$ is an input signal data at sampling time r, $w(*)$ is a window function, and $\hat{W}_N^{-kr}$ are the ST-DFT Hilbert transform operators defined as follows,

$$\hat{W}_N^{-kr} = \begin{cases} \exp\{-j(2\pi kr/N+\pi/2)\}, \\ \quad \text{if } 0<k<N/2 \\ \exp[j\{2\pi(N-k)r/N+\pi/2\}] \\ \quad = \exp\{-j(2\pi kr/N-\pi/2)\}, \\ \quad \text{if } N/2<k<N \\ 0, \text{ if } k=0,\ N/2. \end{cases} \quad (3)$$

Output signals of ST-DFT Hilbert transformers at sampling time n, $y(n)$, are given by the inverse ST-DFT as follows,

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1}\hat{\phi}_k(n)W_N^{kn},$$

$$W_N^{kn} = \exp(j2\pi kn/N) \quad (4)$$

The window function, $w(*)$, in Eq. 2 plays roles of the decimation filter in the ST-DFT.

3. Constrict Conditions

Existence of the ST-DFT gives the decimation filter, $w(*)$, the constrict conditions as follows,

$$w(qN)=\begin{cases}1, & \text{if } q \text{ is zero,}\\ 0, & \text{if } q \text{ is non-zero integer.}\end{cases} \quad (5)$$

Here, N is number of frame samples.

Nyquist filter, $h(n)$,

$$h(n) = \sin(n\pi/N)/(n\pi/N),$$
$$-mN/2 \leqq n \leqq mN/2. \quad (6)$$

is, hereafter, employed as decimation filter of ST-DFT, because $h(n)$ also satisfies Eq. 5. Here, m means the window frame number.

Figure 1 (a) shows that the product sum of every i-th point of frame converge to unity when frame number of Nyquist filter increases. The frequency responses are shown in Fig. 1 (b) as taking a frame number of Nyquist filter as parameters. The infinite Nyquist window function in which the product sums of every i-th point becomes unity has ideal frequency response as shown in Fig. 1 (b). Such, decimation filters as its prodict sum of every i-th point being unity are seemd to act as ideal filters.

Therefore, the constrict condition of being unity gives decimation filter, $w(n)$, following condition,

$$\sum_{l=-m/2}^{m/2-1} w(lN+i) = 1,$$
$$i = 0, 1, 2, \cdots, N-1. \quad (7)$$

4. Design Specifications

The optimization of decimation filter is required to do under both constraints Eqs. 5 and 7.

Initial valueX_0 of the optimization set to equal those of truncated Nyquist filter.

$$X_0 = [h(-mN/2)h(-mN/2+1) \cdots h(0) \cdots h(mN/2)]^T \quad (8)$$

In the first step, Nyquist filter is optimized by the steepest gradient method under only the constraint of Eq. 5. In the second step, the discontinuous points of this optimized filter are modified to hold continuity, because the ideal decimation filters are continuous functions.

In both steps, optimization of the filter is performed as taking the parameters of decimation filter during $-N$ and N as dependent variable to hold the constrict condition of Eq. 7.

The steepest gradient method is given as follows,

$$X_{i+1} = X_i - \delta \, \text{grad} X_i \quad (9)$$

Here,

$$\text{grad}X = [J(X+\Delta) - J(X)]/\Delta \quad (10)$$

And,

$$J(X) = \max \mid H(\Omega) - H_d(\Omega) \mid \quad (11)$$

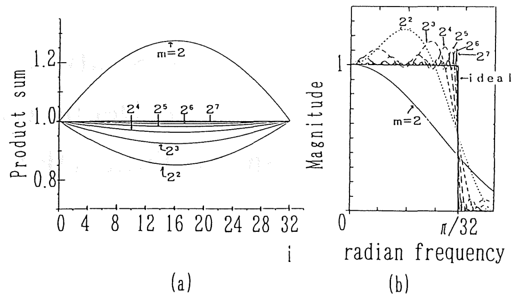

Fig. 1. Characteristics of Nyquist filter with frame sample number $N=32$ and m frame number. (a) product sum of i-th point. (b) Frequency response.

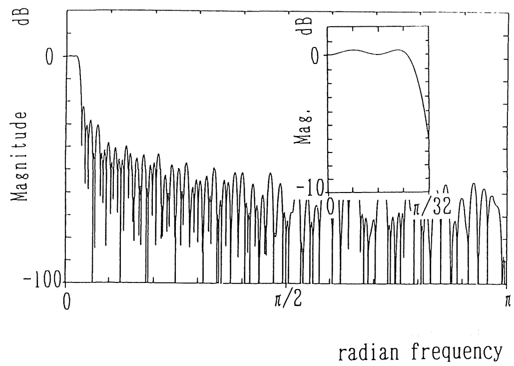

Fig. 2. Frequency response of the new optimized decimation filter. ($N=32$, and $m=8$).

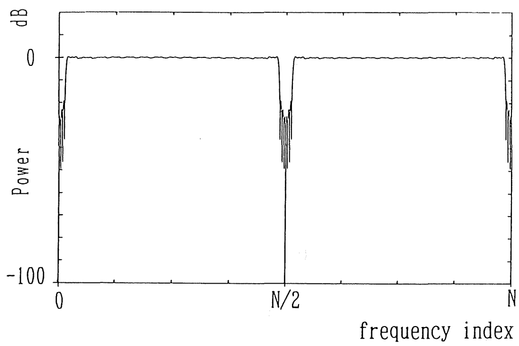

Fig. 3. Power sepctrum of unit sample response.

Where, X_i is a parameter vector at repeating $i-1$ times, $J(X)$ is an evaluation function in which the maximum ripple is taken as the function value, $H_d(\Omega)$ is an expected value of ripples on the passband, $H(\Omega)$ is frequency response of the filter on the passband, δ is a stepping quantity, and Δ is minute variation.

5. Comparison

Figure 2 shows frequency response of the new optimized decimation filter. This filter satisfies the constraints of Eqs. 5 and 7, and the biggest ripple amplitude is smaller than Nyquist filter.

Figure 3 shows power spectrum of unit sample response, when the new optimized decimation filter is employed in the ST-DFT Hilbert transformers. This optimized decimation filter has the features that the biggest ripple amplitude on the passband is reduced, while the small ripples are dispersed over all of domain.

This result is considered to rely on selection of dependent parameters to satisfy the constraint of Eq. 7. Further study will be focused on the selection of dependent parameter in Eq. 7.

References

(1) M. Kishi: "A proposal of short time DFT Hilbert transformers and its configuration", Trans. IEICE, E71, 5, pp. 466-468, May 1988.

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