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Size Effect on Expression for Stress-Strain Curves of Concrete and Its Application for Moment-Curvature Relationships of Reinforced Concrete Beams

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寸法効果を考慮したコンクリートの応力度ーひずみ度曲線の表示式とRC梁のモーメントー曲率関係への適用

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A series of experiments are carried out to examine the effect of such parameters as size effect on compressive strength and stress-strain curves of concrete, and size effect on the moment-curvature relationships of reinforced concrete beams. Based on the experimental data, the stress-strain curves of concrete made with different size of specimens are proposed and its application for moment-curvature relationship of reinforced concrete beams is discussed. The plastic deformational capacity of reinforced concrete beam is remarkably affected by the size of beam specimen and size of aggregate.

1. INTRODUCTION

The stress-strain curves in the stress ascending and descending portion of concrete are markedly affected by the size of concrete specimen and diameter of aggregate. The moment-curvature relationships in plastic deformation range of reinforced concrete beams under uniform bending moment are substantially affected by the stress-strain curves of concrete in the compressive zone of beams. Study on inelastic deformational behavior of model concrete is particularly important to discuss the moment-curvature relationship in plastic deformation range of model reinforced concrete beams with different size of specimen. Tests and theories on size effect of compressive strength of concrete have been examined by many researchers while no generally accepted numerical expression of model concrete with different size.

The objectives of the present study are to examine the inelastic deformational behavior and the numerical expression of the inelastic stress-strain relationship of model concrete prisms and to examine experimentally the plastic rotational capacity of reinforced concrete simple beams made with different size of specimens and to compare the moment-curvature relationships obtained by the experiment and the analysis with the numerical stress-strain relations of model concrete. The authors have been carried out the concrete prism compressive tests and reinforced concrete simple beam bending tests and examined the inelastic deformational behavior of model concrete prisms with different size of specimen and different size of aggregate up to a large compressive strain and moment-curvature relationship of model beams under flexural load.

2. EXPERIMENTAL PROCEDURES

2.1 OUTLINE OF EXPERIMENT

The outline of concrete compressive test is shown in Table 1. The variables in the concrete compressive test were the water-cement ratio (W/C=0.45, 0.60, and 0.70), lateral size of specimen (S=D=4.46, 5.55, 7.25, 9.68, 12.48, and 15.00 cm), and maximum diameter of aggregate (ϕ=10, 15, 20, 25, and 30 mm). Six kinds of prism compressive specimens having square section with different sizes were made. The sizes of section (D×D) were 4.46×

Table 1 Outline of expeliment

Notation of specimen	Notation of size of specimen	Size of specimen Width D (: S) (cm)	Height H (cm)	Water-cement ratio W/C (%)	Size of aggregate ϕ (mm)	Number specimen
60—PR—C—9—ϕ20	PR— 4	4.46× 4.46	13.48	45	15, 25	
‡1	PR— 5	5.55× 5.55	16.65			
‡2	PR— 7	7.25× 7.25	21.75	60	10, 15, 20	20
‡3	PR— 9	9.68× 9.68	29.04		25, 30	
‡4	PR—12	12.48×12.48	37.74			
‡5	PR—15	15.00×15.00	45.00	70	15, 25	

‡1 : Maximum size of aggregate ‡2 : Size of specimen ‡3 : Uniaxial compressive test
‡4 : Prism specimen ‡5 : Water cement ratio

Table 2 Mix proportion of concrete

Water-cement ratio (%)	Specimen series	Size of gravel (mm)	Unit weight(kg/m^3) Water	Coment	Sand	Gravel	Sand pe-rcentage (S/A)(%)	Slump (cm)
60	10Ag.series	10-5	191	319	737	1092	41	15.0
	15Ag.series	15-5	186	310	756	1094	42	15.0
	20Ag.series	20-5	182	304	738	1133	40	15.0
	25Ag.series	25-5	178	297	723	1162	39	15.0
	30Ag.series	30-5	174	290	706	1196	38	15.0

4.46, 5.55×5.55, 7.25×7.25, 9.68×9.68, 12.48×12.48, and 15.00x15.00 cm having height to lateral dimension (h/D) ratio of 3.0. Five kinds of concrete mix proportions having each different size of maximum diameter of aggregate were used in the concrete compressive test.

2. 2 FABRICATION AND CURING OF SPECIMENS

Ordinary portland cement, Tenryu river sand (maximum size : 5 mm) and Tenryu river gravel were used for concrete. Water-cement ratio (w/c) of concrete were 45, 60, and 70% by weight. Five kinds of size of maximum diameter of aggregate (sieve dimension : 10, 15, 20, 25, and 30 mm) were prepared for each concrete series. Mix proportion of concrete is shown in Table 2. The slump of concretes was designed to be 15 cm for all batches. Prism spesimens were cast in steel models in lateral position. Each concrete specimens were fabricated carefully, so as to place the aggregate as inclusion in concrete molds with equal density. Specimens were stored in a labolatory (20±2°C, relative humidity 60±5%) until testing. The tests of concrete were carried out at the age of about six weeks.

2. 3 METHODS OF LOADING AND MEASUREMENT

All the specimens were loaded under the constant strain rate of about 0.1%/min. by a stiff compressive testing machine. Longitudinal strains were measured by two displacement transducers attached to the specimen having the gage length of 2D. The complete stress-strain curves were recorded by an X-Y recorder up to its specified strain (ε = 1.2%).

3. TEST RESULTS AND DISCUSSION

3. 1 SIZE EFFECT ON COMPRESSIVE STRENGTH

Fig.1 shows the relationship between compressive strength and size of specimens for the concrete of W/C=60%. Compressive strength of prism specimens increases as the size of specimens becomes larger, but as may be seen from figure, the curve has a peak at S=9.68cm and there after no increase in the strength is observed and decrease

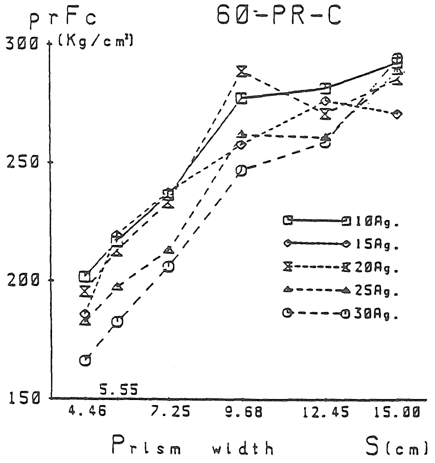


Fig. 1 Compressive strength

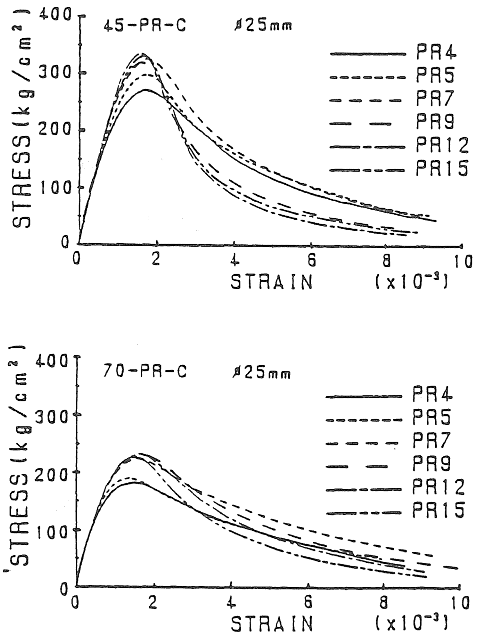


Fig. 2 Effect of W/C ratio on stress-strain curves

with increase in size of specimens S in the range where S is 9.68-15.0cm. On the other hand, the compressive strength decreases in parallel with increase in size of aggregate in concrete. These tendencies were already confirmed by the auther's previous study (1).

3.2 SIZE EFFECT ON STRESS-STRAIN CURVE

Fig.2 shows the effect of the water-cement ratio on stress-strain curves. It is obvious from the

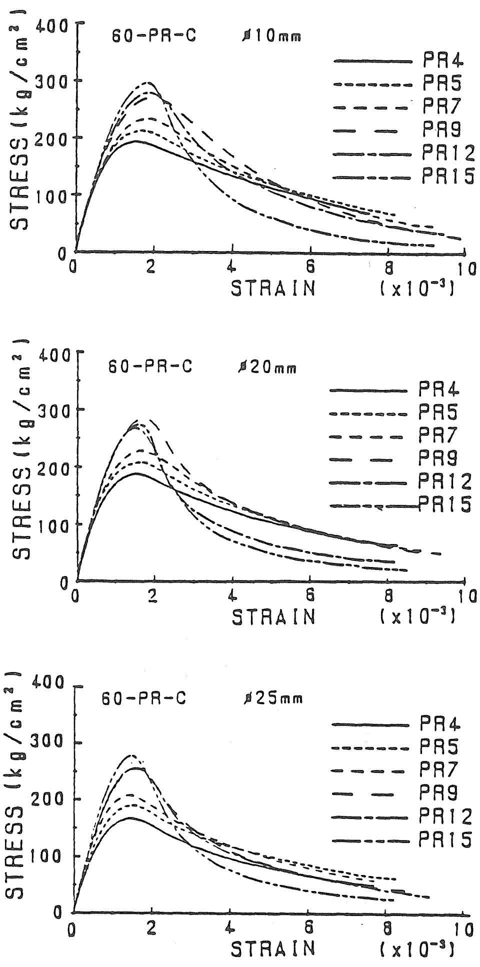


Fig. 3 Effect of size of specimen on stress-strain curves

figure that the slope of the stress-strain curves in the stress descending portions are larger when the water-cement ratio becomes smaller and compressive strength becomes larger.

Fig.3 shows the effect of the size of specimen with different size of aggregate on stress-strain curves. The shapes of stress-strain curves are markedly affected by the size of specimens. When the compressive strength becomes larger, the descending portion of the stress-strain curve becomes steeper with the increase in the size of specimen. Furthermore, the ductility of stress-strain curves in the descending portion decreased with the increase of the size of specimen, compared with the specimen series having equal strength level.

Table 3 Expression for stress-strain curves

Normalized stress-strain curves ($\bar{S}-E$ curves)

(I) Stress ascending portion

$$\bar{S}=\frac{Na \cdot E}{Na-1+E^{Na}} \cdots\cdots\cdots\cdots (1)$$

$\bar{S}=\sigma/\sigma_0$, $E=\varepsilon/\varepsilon_0$

$Na=1+a(\sigma_0/100)^b$

$a=0.57$, $b=1.0$

(II) Stress decending portion

$$\bar{S}=\frac{Nd \cdot X}{Nd-1+X^{Nd}} \cdots\cdots\cdots\cdots (2)$$

$\bar{S}=\sigma/\sigma_0$, $E=\varepsilon/\varepsilon_0$

B, Nd : Empirical constant

Where : $X=B(E-1)^m+1$, $E \geqq 1$

m : Empirical constant $m=0.8$

$$B=\frac{0.023\phi \cdot S+0.199\phi+0.0431S+0.0109}{0.21\phi+8.525\times10^{-3}S+0.805} \cdots\cdots\cdots\cdots (3)$$

$$Nd=\frac{0.1097S-2.0\times10^{-3}\phi+2.854}{0.21\phi+8.525\times10^{-3}S+0.805} \cdots\cdots\cdots\cdots (4)$$

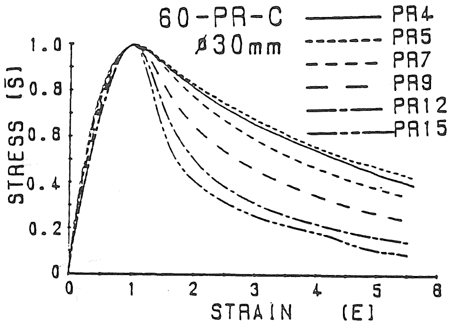


Fig. 4 Normalized $\bar{S}$-E curves (60-PR-C series)

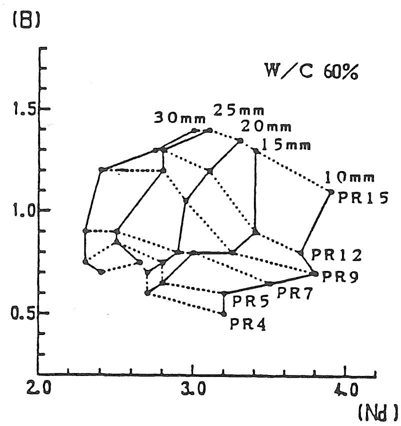


Fig. 5 B-Nd relation (m=0.8)

4. NUMERICAL EXPRESSION FOR STRESS-STRAIN CURVE

The equation for the stress-strain curve proposed by Popovics' (2) for the stress ascending portion was used in this investigation. This equation (1) is shown in Table 3. Among the proposed equations, Popovics' equation is considered to be the most applicable to the stress-strain curves of various concretes.

Hatanaka et al. (3) proposed the equation to express the descending portions of the stress-strain curves of various concretes. This equation (2) is shown in Table 3. Equation (2) is considered to be the most applicable to the normalized stress-strain curves ($\bar{S}$-E curves, $\bar{S}=\sigma/\sigma_0$, σ_0 : compressive strength, $E=\varepsilon/\varepsilon_0$, ε_0 : strain at the compressive strength) of the various concretes. One example of the $\bar{S}$-E curves is shown in Fig.4.

In this investigation, Equation (2) was used for expressing the stress-strain curve of concrete in the stress descending portion by setting the constsnts of B, Nd, and m according to the experimental data, respectively.

Fig.5 shows the relationship between B and Nd for the concrete of W/C=60%, where the value of m is set to 0.8. It can been seen from Fig.5 that size of specimen and size of aggregate are considerably influenced by the relationship between Nd and B. Fig.5 shows that there are deviations from Nd-B relation in the range of $S/\phi < 3.0$. In case of the

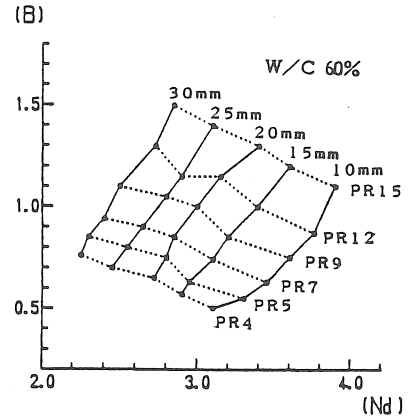


Fig. 6 Calibrated B-Nd relation (m=0.8)

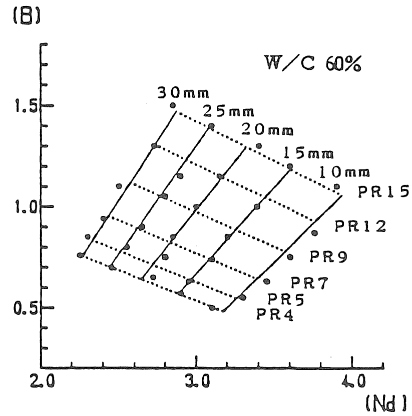


Fig. 7 B-Nd relation calculated by Eq. (3) and (4) (solid and dotted line : calc.)

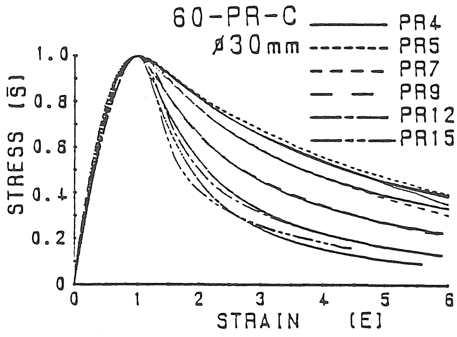


Fig. 8 $\overline{S}$-E curves calculated by the constant B and Nd in Fig.5

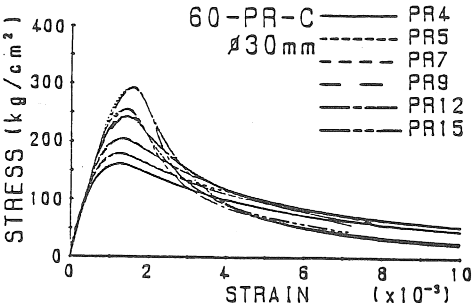


Fig. 9 Stress-strain curves calculated by Fig.8, Eq. (1) and Eq. (2) (dotted line : calc.)

use of model concerte made with $S/\phi < 3.0$ in experiment, points to which special attention should be paid are increase in deviation of stress-strain curves.

Fig.6 shows the relationship between B and Nd for the concrete of W/C=60% being adjusted by varing the parameter (size of specimen and size of aggregate).

The authors carried out simple beam bending test varying the dimensions made with the same mix proportion as this test program and examined experimentally the moment-curvature relationship. The equation (3) and (4) shown in table 2 were obtained, where empirical constant B and Nd are expressed by using size of specimen S (cm) and size of aggregate (cm), to compare the moment-curvature relationships obtained by the beam experiment and analitical results using the proposed stress-strain relations in equation (1), (2) and (3), (4).

Fig.7 shows the B-Nd relations, where solid and dotted lines are calculated by equation (3) and (4). Equation (3) and (4) can be fitted well enough to the empirical B-Nd relations in Fig.5.

Fig.8 shows the relationship between the variation of relative stress $\overline{S}$ and relative strain E calculated by the constant B and Nd in Fig.6 and equation (1), (2). Fig.9 shows the fitness of the proposed equation for the experimental stress-strain curves for the concrete of W/C=60%.

5. PREDICTION OF MOMENT-CURVATURE RELATIONSHIPS OF MODEL RC BEAMS

The moment-curvature relationships obtained from the experiment and from the analysis with the proposed stress-strain relations in equation (3) and (4) were compared, to discuss the applicability of the proposed stress-strain relations for the plastic rotational behavior of the critical section of model reinforced concrete beams.

The sizes of beam section tested (b×h, cm) were 4.46×8.92, 7.25×14.50, 9.68×19.36, and 12.48×24.96 having its effective depth d=0.9h. Fig.10 shows outline of beam specimen. Stirrups were placed closely to prevent shear failure. These beams were made with the same mix proportions as the previous test program for the concrete of W/C=60%. The moment-curvature relationships of model beams subjected to two concentrated loads acting at the third points (uniform moment span=3h) were measured.

Fig.11 shows the moment-curvature relationships at critical section (curvature measurement length=2h) of model beams (W/C=60%, ϕ=25mm, p_t=2.80%). Fig.12 shows the stress-strain curves calculated by equation (1), (2) and (3), (4), and concrete prism compressive strength and strain at the compressive strength obtained from the experiment for the concrete made with ϕ=25mm. Fig.13 shows the moment-curvature relationships at the critical section of the model beams (p_t=2.80%) calculated by the stress-strain curves shown in Fig.12. It is observed from the figure that the plastic rotational behavior of reinforeced concrete beams are affected by the size of beam specimens, because of the size effect of the strength and stress-strain relationships of concrete in the compressive zone of model beams. Fig.13 shows that close agreement between observed and calculated curvature is obtained except for the beam of b=4.46cm.

Fig.14 shows the analitically obtained moment-curvature curves calculated by Eq. (1), (2) and (3), (4) for the beams made with ϕ=20mm,

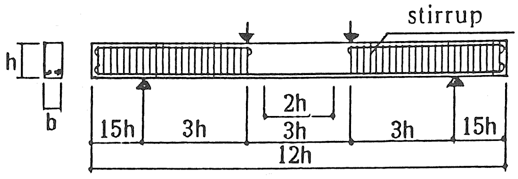


Fig. 10 Outline of beam specimen

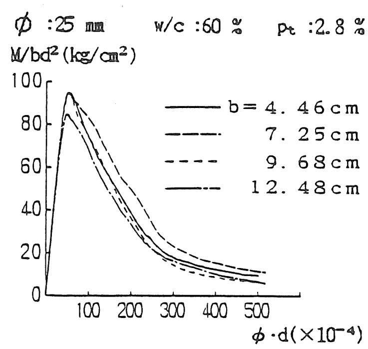


Fig. 11 Moment-curvature curves obtained from RC beam bending test

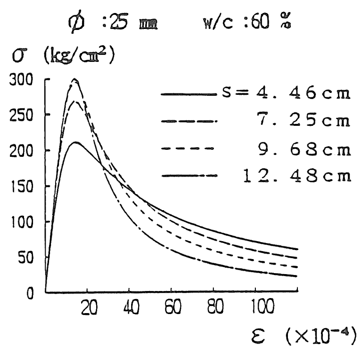


Fig. 12 Stress-strain curves calculated by Eq. (1) (2) and (3), (4)

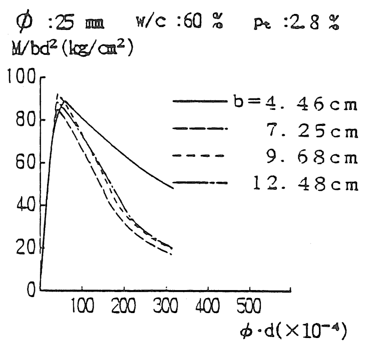
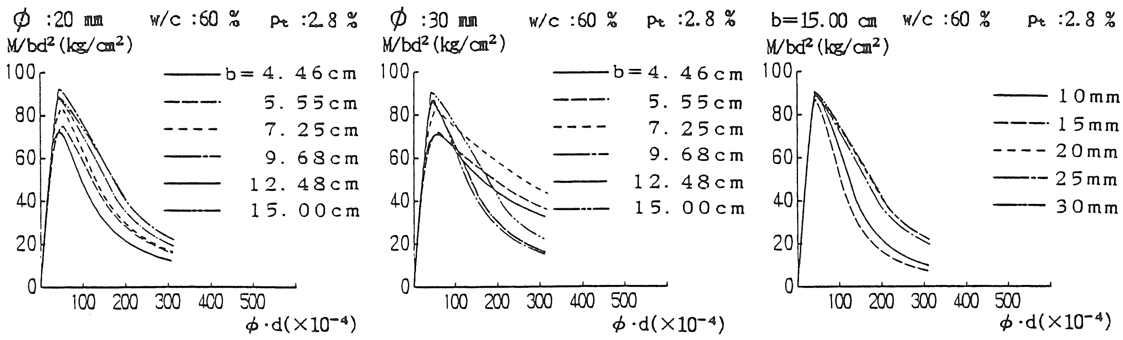


Fig. 13 Moment-curvature curves calculated by stress-strain curves shown in Fig.11



(a) ϕ=20mm series (b) ϕ=30mm series (c) b=15cm series

Fig. 14 Analitically obtained moment-curvature curves calculated by Eq. (1) (2) and (3), (4) (p_t=2.80%, σ_{sy}=4000kgf/cm²)

30mm and b=15.0cm (p_t=2.80%, σ_{sy}=4000kgf/cm^2). Fig.14 (b) shows that the calculated curves of S/ϕ < 3.0 indicate too ductile comparing the calculated with the experimental. Fig.14 (c) shows the calculated moment-curvature curves of reinforced concrete beam (b=15.0cm) made with the different size of aggregates. Because of the use of the concrete of S/ϕ > 3.0 in Fig.14 (c), the phenomenon shown in Fig.14 (b) (b=4.46−7.25cm) are not observed.

The followings can be drawn on the plastic rotational capacity of reinforced concrete beam from the above and other numerical analyses. The plastic deformational capacity of reinforced concrete beam is remarkably affected by the size of beam specimen and size of aggregate. The use of the concrete of S/ϕ < 3.0 is recommended to avoid.

6. CONCLUSION

Size effect on inelastic deformational behavior of concrete and reinforced concrete beam was discussed in this paper experimentally and analytically. The following results were obtained.

(1) Equation (2) proposed by Hatanaka et al' for the stress-strain curves in the stress descending portion was used in this investigation.

(2) The compressive strength becomes larger and the descending portion of the stress-strain curves becomes steeper with the increase in the size of concrete specimen.

(3) The ductility of stress-strain curve in the stress descending portion decreased with the increase of the size of concrete specimen, comparing with the specimen series having the equal strength level.

(4) Size of concrete specimen and size of aggregate markedly influence the relationship between empirical constant Nd and B in equation (2), where the value of m is set to 0.8. These are shown in Fig. 6.

(5) The plastic rotational behaviors of reinforced concrete beams are affected by the size of beam specimens, because of the size effect of the concrete in the compressive zone of model beam.

(6) The calculated moment-curvature cueve of reinforced concrete beam made with the concrete of S/ϕ < 3.0 indicate too ductile compared with the experiment.

(7) The plastic deformational capacity of reinforced concrete beam is remarkably affected by the size of beam specimen and size of aggregate. The use of the concrete of S/ϕ < 3.0 for the beam experiment under flexure should be avoided.

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