Study on Probability Distribution and Size Effect of Compressive and Tensile Strength of Concrete

Sachio KOIKE and Zennosuke KATO

コンクリートの圧縮強度と引張強度の確率分布と寸法効果に関する研究

小 池 狹千朗 • 加 藤 善之助

In the case of calculating the probability distribution of strength of reinforced concrete members by Monte Carlo simulation by the computer, it is very important to sample the probability distribution of mechanical properties of cach materials. This study examined the probability distribution and size effect of compressive strength of three kinds of concrete cylinders and four kinds of concrete prisms and direct tensile strength of four kinds of concrete prisms and splitting tensile strength of three kinds of concrete cylinders, using four kinds of concrete mix proportions having each different maximum sizes of aggregates.

The experimental value of strength shows the probability distribution quite close to the straight line when plotted either on weibull probability paper or on normal ones, but some values are slightly apart from the straight line near the maximum and minimum experimental values. But, experimental values of coefficient of variation CV of strength of concrete show the lower values than theoretical ones indicated by the formula $CV = \sqrt{\frac{\Gamma(1+\frac{2}{\beta+1})}{\Gamma^2(1+\frac{1}{\beta+1})} - 1}$, wcere, β is parameter related to properties of concrete and Γ is gamma function. Therefore, the probability distribution of its strength cannot always be expressed by the weibull distribution.

Compressive strength of concrete decreases with decrease in size of specimen, both in prism and cylinder specimen, and with increase in size of aggregate. On the other hands, tensile strength of concrete reaches the top at specimen size of 10cm both in prism and cylinder specimens, but it rather decreases when specimen size becomes smaller than 10cm.

1. INTRODUCTION

Study on valiability of streogth and deformation of reinforced concrete members is particularly important to discuss the structural safety. Recently, papers simulated these variabilities by Monte Carlo method have been reported by many researchers 〔Ref. 1 - 7〕. In the case of calculating size effects and the probability distribution of flexural and shear strength of reinforced concrete members by Monte Carlo simulation by the computer, sampling the probability distribution of compressive and tensile strength of concrete being one of the materials such as reinforcing bars and concrete composing reinforced concrete members has a very important meaning.

Tests and theories on size effect of compressive strength of concrete have been examined by many researchers since 1925 [8—18] , while no generally accepted theories and experimental equations for predicting size effects exist at present. Generally, it is well known that the probability distribution of concrete strength follows a Weibull distribution when concrete shows a perfectly brittle fracture mode [19, 20,15 and 21], but Tanigawa, Yamada and Yokoyama showed in their experiments that the probability distribution of concrete strength could not always be expressed by a Weibull distribution [Ref.22] . Recently, making allowance for these facts, some of the failure probability models are proposed for the materials which can not be expressed by Weibull distribution [Ref. 23, 21, 18, 24 and 25] , and it is expected to investigate the validity of these failure models.

On the other hand, tensible strength of concrete has a significant influence on several important physical properties such as flexural and shear cracking load and creack patterns, shear strength and bond strength of deformed bars in reinforced concrete members [Ref. 18] . Many researchers have pursued their studies on tensile strength of concrete using many kinds of test arrangment [Ref. 26-33, 34-39, 40-45]. But, researches refering to size effect as well as to the probability distribution of tensile strength of concrete in direct or indirect tensile test are not sufficient at present [Ref. 34-39]. It is necessary to pursue the reseaches on these probability disrtibution in order to simulate the probability distribution of flexural and shear cracking load and ultimate shear strength of reinforced concrete members by Monte Carlo technique.

This study examined size effects and the probability distribution of direct tensile strength of concrete with four kinds of maximum size of aggregate by lazy tongs grips method [Ref. 40] and indirect tensile strength by cylinder splitting test, fabricating four kinds of prism specimens for direct tensile test and three kinds of concrete cylinders for splitting tensile test, and examined those of compressive strength of concrete by prism and cylinder specimens, and offered data to simulate occurence of the probability distribution of concrete strength used.

2. EXPERIMENTAL PROCEDURE

The experiment was carried out in accordance with the test program as shown in Table 1.

Table 1 Outline of experimen.

concrete	w/c	prism specimen				cylinder specimen				prism specimen		Cubic specimen	
		Compressive test		Direct tensile test		Compressive test		Spring tensile test		Test of modulus of rupture		Compressive test	
		Size (cm)	No.of spec.	Size (cm)	No.of spec.	Size(cm)	No.of spec.	Size(cm)	No.of spec.	Size(cm)	No.of spec.	Size(cm)	No.of spec.
10Ag.series		4.46×4.46×13.4		4.46×4.46×13.4		φ7.5×15		φ7.5×15		10×10×40		10×10×10	
15Ag.series	60	7.25×7.25×21.8	15	7.25×7.25×21.8	30	φ10×20	20	φ10×20	20		3		6
20Ag.series		9.68×9.68×29.0		9.68×9.68×29.0									
25Ag.series		15.0×15.0×45.0		15.0×15.0×45.0		φ15×30		φ15×30		15×15×53		15×15×15	

(1) Test Specimen

Prism tensile specimens are plain concrete prisms reduced central parallel section with enlarged ends and without reinforced ends, and prism compressive specimens have its height to lateral dimension (h/D) ratio 3.0. These specimens are shown in Fig. 1. Three kinds of concrate cylinders, $\phi 7.5\times 15$, $\phi 10\times 20$ and $\phi 15\times 30$ cm were casted for compressive test and splitting tentile test, where ϕ is diameter of cylinder.

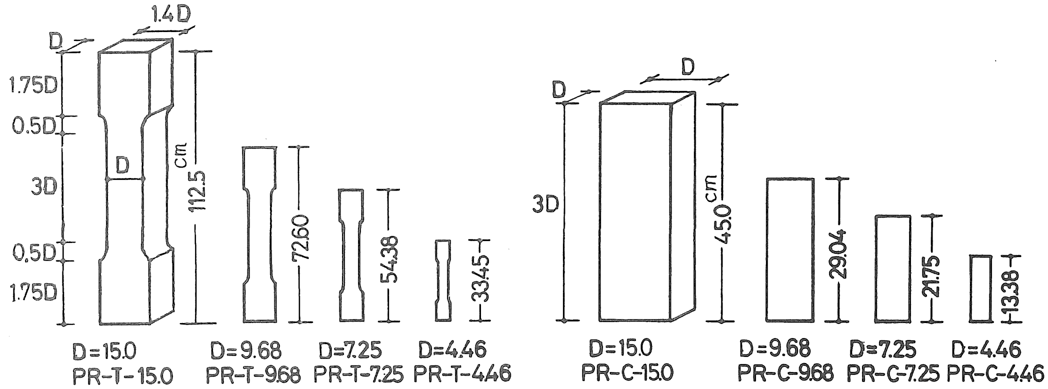

Fig. 1 Outline of prism specimens in compressive and direct tension test.

The variables in the experiment are as follows: four different sizes of tension prism specimens (d=4.46, 7.25, 9.68 and 15.0 cm) and three different sizes of splitting and compressive cylinder specimens ($\phi=7.5$, 10.0 and 15.0 cm) were also prepared to obtain the properties of concrete used.

(2) Fablication and Curing of Specimen

Ordinary portland cement, Yahagi river sand and Tenryu river gravel were used for concrete. The properties of aggregates used are shown in Table 2. Mix proportions of four kinds of concrete are shown

Table 2 Properties of aggregate.

Kind of concrete	Kind of aggregate	Aggregate size (mm)	Specific gravity	Water absorption 24hrs.(%)	Fineness modulus
10Ag.series	river gravel	10~2.5	2.65	0.99	5.57
	river sand	1.2~	2.58	1.56	2.95
15Ag.series	river gravel	15~5	2.65	0.93	6.25
	river sand	1.2~	2.58	1.56	2.95
20Ag.series	river gravel	20~5	2.66	0.90	6.57
	river sand	2.5~	2.58	1.50	2.95
25Ag.series	river gravel	20~5	2.66	0.90	7.00
	river sand	2.5~	2.51	1.80	2.58

Table 3 Mix proportion of concrete.

Kind of concrete	Size of gravel (mm)	Water (kg/m³)	Cement (kg/m³)	Sand (kg/m³)	Gravel (kg/m³)	s/a (°/vl)	Design		Measured	
							Air(°/vl)	Slamp(cm)	Air(°/vl)	Slamp(cm)
10Ag.series	10〜2.5	230	383	659	1015	40	1.0	15	1.25	14.2
15Ag.series	15〜	220	367	708	1004	42	1.0	15	0.66	16.0
20Ag.series	20〜	210	350	759	996	44	1.0	15	0.75	15.4
25Ag.series	25〜	210	350	739	996	44	1.0	15	1.80	14.3

in Table 3 and water-cement ratio (w/c) of concrete was 60% by weight. Four kins of maximum sizes of aggregate (sieve dimension=10, 15, 20 and 25 mm) were prepared as inclusion, respectively.

Prism specimens for compressive test having steel mold at both ends were cast in wood mold holizontally. Each concrete specimens were fabricated most carefull, so as to place the aggregates as inclusion in concrete molds with equal density. Cylinder specimens stored in a labolatory during 48 hours after casting, then they were remolded and cured in moisture room at a temperature of 20°±1℃ and a relative humidity of over 80% until just before the test during six weeks.

(3) Method of Loading and Measurement

The loadings and supports were accomplished with the same size of plates as specimens both prisms and cylinders and spherical seats molded to the same scale as the test specimen used in comperessive test. Direct tensile test technique was used for specimens with enlarged ends to which load was applied purely by friction using four kinds of lazy tongs grips shown in Fig. 2 [Ref.40—45]. Generaly, it is more suitable for testing a large number of different sizes of specimens. Total of 240 prism specimens were tested in compressive test, 210 cylinders in compressive, 480 prisms in direct tensile and 270 cylinders in splitting, respectively.

Longitudinal strain (ε) was measured by two strain gauge type deformation transformers attached to the specimen (meansured length=1.8D) in compressive test and was measured by wire resistance strain gauges (gauge length=60 mm) in direct tensile test.

3. TEST RESULT AND DISCUSSION

Table 4.1 and 4.2 show the actual dimensions of specimen after removing mold and show the test results, where "size of specimen" indicates mean value.

(1) Fracture Distribution

Direct tensile test specimens have a hight of three times its depth (d) in central parallel test length. The incidence of fracture was very greater in the top parts of test length in the case of PR-T-15.0 series specimens,and was very greater in the central 2d part in the case of another sizes of specimens. Johnston and

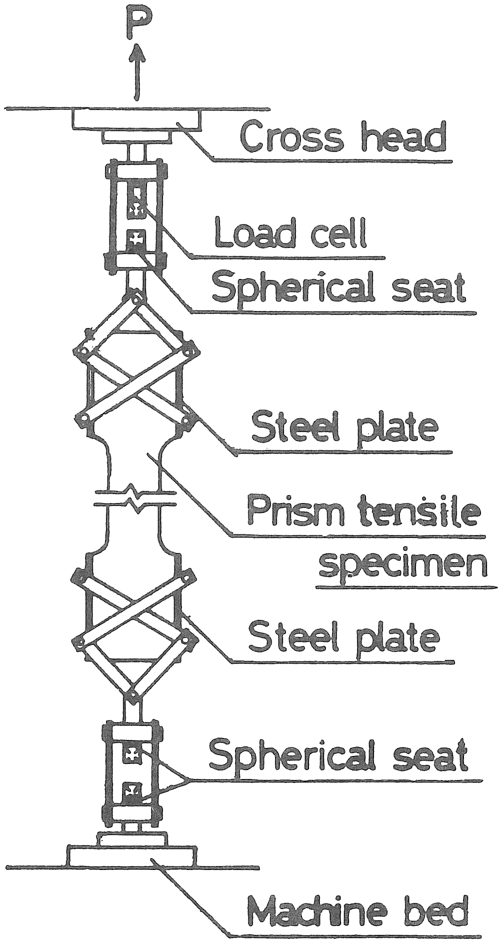

Fig. 2 Test arrangement in direct tensile test using lazy tongs grips.

Sidewell reported that the incidence of fracture, althorgh reasonably uniform, is norticeably greater in the upper part of test length [Ref. 43] .

(2) Probability Distribution of Strength

The relations between non-failure prabability 1n (-1n(1-P)) and strength 1n(F) obtained by four kinds of test are shown in Fig.3.1. P (failure probability corresponding to the strength of No. n counted from the smallest one) was calculated by the following formula.

Table 4.1 Test result (1).

Kind of concrete	Notation of specimen	No. of specimen	Size of specimen Width (cm)	Depth (cm)	Length (cm)	Rang of stress (kg/cm²)	Prism compre. strength M (kg/cm²)	SD (kg/cm²)	CV (%)	Weibull dist. r	LN(-LN(1-P))	Slope	β	Normal dist. r	μ	σ	Log-normal dist. r	μ	σ
10 Ag. series	10-PR-C-4.46	14	4.693	4.479	13.38	249~356	294	30.3	10.30	0.956	9.92X- 56.83	9.92	8.92	-0.977	295	36.5	-0.986	5.68	0.121
	10-PR-C-7.25	13	7.453	7.252	21.75	199~276	249	21.8	8.79	0.986	11.20X- 62.25	11.20	10.20	-0.973	249	26.6	-0.961	5.51	0.113
	10-PR-C-9.68	15	9.749	9.688	29.04	233~286	260	15.1	5.81	0.988	17.91X-100.07	17.91	16.91	-0.992	260	17.8	-0.992	5.56	0.069
	10-PR-C-15.0	14	15.128	15.115	45.00	264~305	286	11.3	3.96	0.969	25.71X-145.89	25.71	24.71	-0.979	286	13.6	-0.982	5.65	0.047
15 Ag. series	15-PR-C-4.46	15	4.620	4.463	13.38	156~283	233	38.4	16.46	0.986	5.91X- 32.68	5.91	4.91	-0.975	234	46.0	-0.961	5.44	0.214
	15-PR-C-7.25	15	7.411	7.235	21.75	222~269	241	12.3	5.10	0.950	9.93X-109.78	19.93	18.93	-0.969	241	14.8	-0.975	5.48	0.060
	15-PR-C-9.68	15	9.806	9.673	29.04	202~238	222	10.7	4.81	0.993	21.67X-117.62	21.67	20.67	-0.992	223	12.6	-0.992	5.40	0.057
	15-PR-C-15.0	13	15.204	15.083	45.00	226~324	275	23.5	8.57	0.954	11.47X- 64.87	11.47	10.47	-0.954	275	29.2	-0.953	5.61	0.107
20 Ag. series	20-PR-C-4.46	15	4.614	4.481	13.38	177~252	210	21.4	10.19	0.942	9.98X- 53.83	9.98	8.98	-0.964	211	26.0	-0.973	5.34	0.120
	20-PR-C-7.25	15	7.391	7.245	21.75	218~263	239	11.9	4.97	0.979	20.86X-114.70	20.86	19.86	-0.987	239	14.0	-0.990	5.47	0.059
	20-PR-C-9.68	15	9.756	9.692	29.04	236~261	250	7.0	2.79	0.971	36.85X-203.97	36.85	35.85	-0.976	250	8.3	-0.980	5.52	0.033
	20-PR-C-15.0	15	15.172	15.105	45.00	230~288	267	14.9	5.60	0.988	18.15X-101.86	18.15	17.15	-0.968	227	18.0	-0.960	5.58	0.070
25 Ag. series	25-PR-C-4.46	14	4.702	4.511	13.38	189~255	227	20.6	9.00	0.987	11.10X- 60.86	11.10	10.10	-0.977	227	24.8	-0.973	5.42	0.113
	25-PR-C-7.25	15	7.425	7.255	21.75	194~268	226	21.1	9.33	0.963	11.05X- 60.37	11.05	10.05	-0.977	226	25.2	-0.984	5.42	0.109
	25-PR-C-9.68	15	9.812	9.693	29.04	209~271	242	18.1	7.48	0.986	13.75X- 75.96	13.75	12.75	-0.985	242	21.5	-0.983	5.49	0.090
	25-PR-C-15.0	15	15.165	15.105	45.00	231~304	257	15.7	6.11	0.852	15.36X- 85.73	15.36	14.36	-0.961	257	21.3	-0.880	5.55	0.078

Kind of concrete	Notation of specimen	No. of specimen	Size of specimen Dia φ (cm)	Length (cm)	Rang of stress (kg/cm²)	Cylinder Compr. strength M (kg/cm²)	SD (kg/cm²)	CV (%)	Weibull dist. r	LN(-LN(1-P))	Slope	β	Normal dist. r	μ	σ	Log-normal dist. r	μ	σ
10 Ag. series	10-CY-C-φ7.5	15	7.491	15.05	236~382	316	41.1	13.00	0.993	7.82X- 45.44	7.82	6.82	-0.990	317	48.5	-0.983	5.75	0.160
	10-CY-C-φ10	15	9.995	20.06	253~351	313	29.4	9.39	0.980	10.60X- 61.35	10.60	9.60	-0.965	313	35.6	-0.955	5.74	0.120
	10-CY-C-φ15	15	14.989	30.12	303~373	329	19.9	6.03	0.944	16.80X- 97.86	16.80	15.80	-0.971	330	23.9	-0.977	5.80	0.071
15 Ag. series	15-CY-C-φ7.5	15	7.489	15.06	251~330	280	20.3	7.25	0.946	14.07X- 79.74	14.07	13.07	-0.961	280	24.7	-0.971	5.63	0.086
	15-CY-Cφ10	15	9.985	20.06	255~336	285	22.0	7.75	0.941	13.15X- 74.79	13.15	12.20	-0.967	285	26.6	-0.977	5.65	0.090
	15-CY-C-φ15	15	14.970	30.10	300~367	330	18.9	5.71	0.968	18.06X-105.26	18.06	17.10	-0.987	331	22.3	-0.991	5.80	0.067
20 Ag. series	20-CY-C-φ7.5	19	7.502	15.08	216~257	239	11.0	4.58	0.992	23.38X-128.55	23.38	22.40	-0.990	239	12.7	-0.990	5.48	0.053
	20-CY-C-φ10	20	9.997	20.06	230~300	265	20.8	7.85	0.982	13.62X- 76.44	13.62	12.60	-0.994	265	23.8	-0.995	5.58	0.090
	20-CY-C-φ15	20	14.974	30.07	284~352	315	21.2	6.73	0.965	15.65X- 90.52	15.65	14.70	-0.984	315	24.6	-0.986	5.75	0.078
25 Ag. series	25-CY-C-φ7.5	20	7.508	15.07	210~296	254	24.8	9.77	0.983	10.75X- 59.99	10.75	9.70	-0.983	254	28.8	-0.979	5.53	0.116
	25-CY-C-φ10	19	9.992	20.07	258~331	285	19.7	6.90	0.946	15.15X- 86.14	15.15	14.10	-0.975	286	23.2	-0.983	5.65	0.079
	25-CY-C-φ15	20	15.008	30.08	243~293	268	13.2	4.91	0.970	21.62X-121.36	21.62	20.60	-0.982	268	15.3	-0.985	5.59	0.057

Kind of concrete	Notation of specimen	No. of specimen	Size of specimen Width (cm)	Depth (cm)	Rang of stress (kg/cm²)	Prism tensile strength M (kg/cm²)	SD (kg/cm²)	CV (%)	Weibull dist. r	LN(-LN(1-P))	Slope	β	Nomal dist. r	μ	σ	Log-nomal dist. r	μ	σ
10 Ag. series	10-PR-T-4.46	26	4.621	4.458	16.0~30.6	24.4	4.06	16.61	0.976	6.05X-19.79	6.05	5.05	-0.970	24.5	4.69	-0.949	3.18	0.213
	10-PR-T-7.25	26	7.450	7.234	21.3~27.8	24.5	1.96	7.98	0.975	13.48X-43.61	13.48	12.48	-0.983	24.6	2.23	-0.980	3.20	0.093
	10-PR-T-9.68	28	9.930	9.626	20.6~28.0	24.2	1.80	7.83	0.985	14.06X-45.29	14.06	13.06	-0.994	23.3	2.13	-0.993	3.19	0.089
	10-PR-T-15.0	28	15.310	15.060	15.6~22.9	18.7	1.61	8.61	0.975	12.79X-37.92	12.79	11.79	-0.985	18.7	1.82	-0.989	2.93	0.097
15 Ag. series	15-PR-T-4.46	23	4.658	4.438	15.3~29.2	21.1	3.57	16.90	0.981	6.35X-19.81	6.35	5.35	-0.987	21.2	4.09	-0.989	3.04	0.195
	15-PR-T-7.25	22	7.481	7.176	20.9~29.3	25.3	2.59	10.21	0.983	10.46X-34.28	10.46	9.46	-0.990	25.4	2.96	-0.987	3.23	0.119
	15-PR-T-9.68	26	9.857	9.560	18.7~28.1	24.0	2.50	10.41	0.994	10.36X-33.40	10.36	9.36	-0.989	24.0	2.83	-0.983	3.17	0.122
	15-PR-T-15.0	27	15.284	14.987	15.8~21.5	18.9	1.52	8.05	0.986	13.56X-40.33	13.56	12.56	-0.990	18.9	1.72	-0.987	2.94	0.092
20 Ag. series	20-PR-T-4.46	17	4.713	4.493	11.5~23.6	17.7	3.73	21.10	0.976	4.72X-13.97	4.72	3.72	-0.980	17.7	4.40	-0.972	2.85	0.264
	20-PR-T-7.25	25	7.265	7.551	16.4~24.9	21.2	2.27	10.69	0.996	10.09X-31.29	10.09	9.09	-0.993	21.2	2.56	-0.986	3.05	0.125
	20-PR-T-9.68	25	9.913	9.533	14.9~26.6	21.0	2.82	13.44	0.980	8.02X-24.88	8.02	7.02	-0.987	21.1	3.22	-0.986	3.04	0.155
	20-PR-T-15.0	27	15.373	14.977	14.5~22.0	18.3	1.77	9.68	0.990	11.30X-33.32	11.30	10.30	-0.992	18.3	2.00	-0.990	2.90	0.111
25 Ag. series	25-PR-T-4.46	22	4.467	4.620	15.5~26.6	20.8	3.57	17.15	0.967	6.23X-19.35	6.23	5.23	-0.982	20.8	4.12	-0.988	3.02	0.196
	25-PR-T-7.25	25	7.458	7.177	14.9~25.8	19.4	2.84	14.66	0.970	7.39X-22.37	7.39	6.39	-0.975	19.4	3.28	-0.982	2.96	0.167
	25-PR-T-9.68	26	9.891	9.526	16.3~26.3	22.2	2.57	11.60	0.991	9.26X-29.17	9.26	8.26	-0.989	22.2	2.91	-0.983	3.09	0.136
	25-PR-T-15.0	25	15.353	15.067	14.6~24.1	19.0	1.86	9.75	0.963	10.97X-32.81	10.97	9.97	-0.964	19.1	2.17	-0.966	2.94	0.114

Kind of concrete	Notation of specimen	No. of specimen	Size of Specimen Dia φ (cm)	Length (cm)	Rang of stress (kg/cm²)	Cylinder splitting strength M (kg/cm²)	SD (kg/cm²)	CV (%)	Weibull dist. r	LN(-LN(1-P))	Slope	β	Normal dist. r	μ	σ	Log-normal dist. r	μ	σ
10 Ag. series	10-CY-SP-φ7.5	25	7.50	15.06	19.5~38.1	26.7	4.24	15.89	0.954	6.94X-23.24	6.94	5.94	-0.962	26.8	4.96	-0.981	3.28	0.176
	10-CY-SP-φ10	25	9.99	24.06	24.0~33.4	29.8	2.59	8.70	0.988	12.15X-41.71	12.15	11.15	-0.972	29.8	3.00	-0.961	3.39	0.106
	10-CY-SP-φ15	25	14.98	30.06	18.5~28.0	24.0	5.73	9.97	0.983	10.44X-33.65	10.44	9.44	-0.966	24.0	2.79	-0.952	3.17	0.124
15 Ag. series	15-CY-SP-φ7.5	24	7.50	15.11	17.4~35.9	26.5	5.17	19.52	0.977	5.56X-18.62	5.56	4.56	-0.987	26.4	5.84	-0.992	3.25	0.222
	15-CY-SP-φ10	25	9.99	20.13	23.3~36.8	30.0	3.87	12.88	0.986	8.38X-28.95	8.38	7.38	-0.993	30.1	4.38	-0.990	3.40	0.149
	15-CY-SP-φ15	25	15.03	30.15	21.1~33.3	27.0	3.01	11.15	0.978	9.73X-32.55	9.73	8.73	-0.985	27.1	3.44	-0.986	3.29	0.128
20 Ag. series	20-CY-SP-φ7.5	19	7.50	15.06	22.0~31.3	26.3	2.50	9.48	0.986	11.15X-36.94	11.15	10.15	-0.989	26.4	2.89	-0.988	3.27	0.111
	20-CY-SP-φ10	19	9.99	20.09	25.0~36.3	29.5	3.36	11.40	0.944	9.20X-31.62	9.20	8.20	-0.970	29.6	3.97	-0.980	3.38	0.130
	20-CY-SP-φ15	19	14.96	30.04	22.4~29.9	25.6	2.05	8.00	0.965	13.19X-43.25	13.19	12.19	-0.986	25.6	2.38	-0.991	3.24	0.092
25 Ag. series	25-CY-SP-φ7.5	19	7.49	15.10	20.4~33.3	27.0	3.40	12.61	0.993	8.39X-28.09	8.39	7.39	-0.995	27.0	3.92	-0.993	3.29	0.148
	25-CY-SP-φ10	19	9.97	20.05	24.6~34.4	29.8	2.88	9.65	0.975	10.94X-37.61	10.94	9.94	-0.987	29.8	3.34	-0.988	3.39	0.112
	25-CY-SP-φ15	20	14.97	30.14	17.4~27.7	23.7	3.00	12.66	0.985	8.19X-26.39	8.19	7.19	-0.982	23.7	3.49	-0.974	3.16	0.154

Table 4.2 Test result (2).

Kind of concrete	Notation of specimen	Size of specimen(cm)	Bending span(cm)	Modulus of rupture(kg/cm²)	Cubic compressive strength(kg/cm²)
10Ag.	10-PR-B-10.0	10.17× 9.97	30	32.9	315
series	10-PR-B-15.0	15.12×15.03	45	35.9	324
15Ag.	15-PR-B-10.0	10.21× 9.96	30	30.7	303
series	15-PR-B-15.0	15.10×15.03	45	34.1	324
20Ag.	20-PR-B-10.0	10.18×10.00	30	33.1	326
series	20-PR-B-15.0	15.15×15.05	45	24.6	303
25Ag.	25-PR-B-10.0	10.00×10.04	30	34.0	318
series	25-PR-B-15.0	15.02×15.13	45	31.2	296

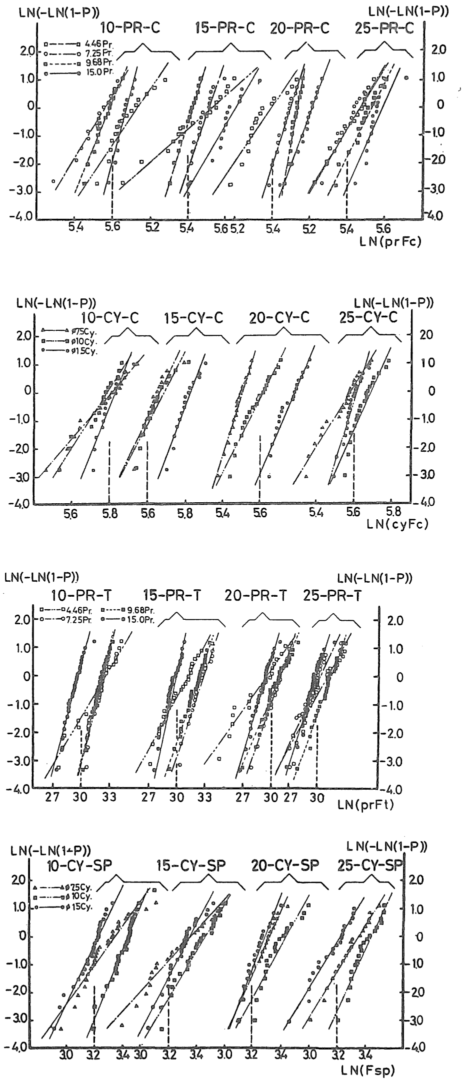

Fig. 3・1 Relation between non-failure probability ln(-ln(1-P)) and strength ln(F) (Weibull distribution).

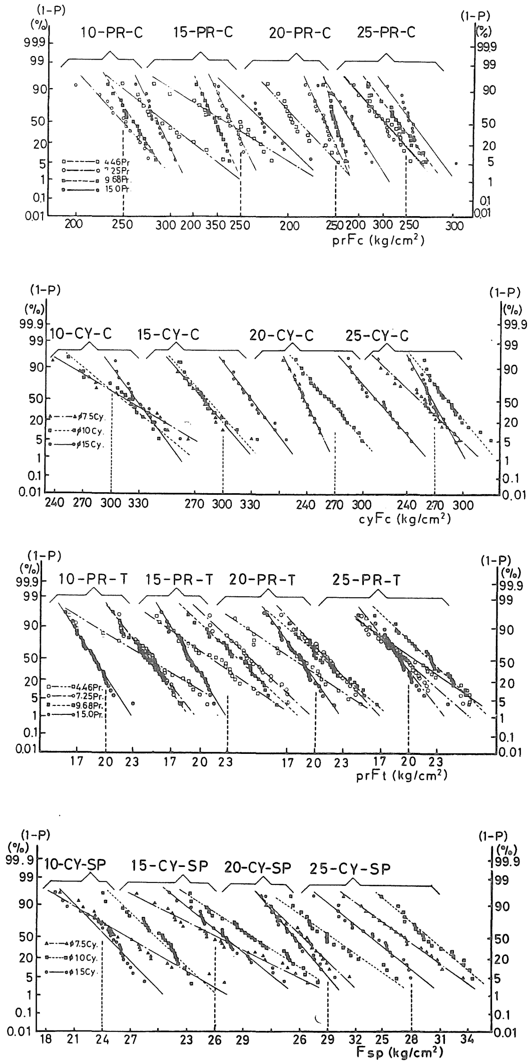

Fig. 3・2 Relation between non-failure probability (1-P) (%) and strength (F) (Normal distribution).

$$P = \frac{1}{N+1} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(1)$$

where, P : failure probability

N : the total number of specimens

When a material shows perfectly bittle fracture mode, the probability distribution of strangth follows a Weibull distribution [Ref. 19, 20, 15 and 21] , the relation between non-failure probability 1n(-1n(1-P)) and the strength 1n(F) is expressed by the straight line expression (Weibull distribution).

$$\text{In}(-\text{In}(1-P)) = \text{In}(A) + (\beta+1)\cdot\text{In}(F) \quad \cdots\cdots(2)$$

where, P: failure probability

β : parameter related to properties of concrete

A : a constant determined by kind of materials, circumstances, size of specimen, etc.

F : strength of specimen

The slope of straight line ($\beta+1$) does not reated to the quantity of tdetects.In this study, it was assumed that the experimental errors were small and distributions of that errors were uniform in the large range of D/d. The straight lines obtained by the least square approximation of experimental values are shown in Fig. 3.1 and 3.2, and the expressions of these straight lines and correlation coefficients are shown in Table 4.1.

The results of tests show correlation coefficient of 0.85-0.99 in prism compressive specimens, 0.94—0.99 in cylinder comperssive, 0.96—0.996 in prism direct tensile and 0.94—0.99 in cylinder splitting. In the case of prism direct tensile test, correlation coefficient is nearest to 1. Fig. 3.2 shows the probability distribution of test results protted on Normal probability paper. Fig. 3.1 and 3.2 show that distributions of experimental values are quite close to the straight line, but some values are slightly apart from the straight line near the maximum and minimum values. This matter requires futher examinations to discuss the structural safety. In order to propose the probability distribution for adequate indication of distribution of exerimental values, it is necessary to accumulate more experimental data.

1) Effect of size of specimen

Fig. 4 and Table 4.1 show the relation between material constant (β) and spacimen size (S). β was calculated by the following formula form "a" (slope of the straight line drawn by the probability distribution of strength calculated by the method of reast squares on the weibull probability paper.

$$\beta = a-1 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(3)$$

Fig. 4 shows the tendency that the value of β increase with increse of S (size of spacimen ; prism spec. : S= prism width, cyclinder spec. : S=cylinder diamater).

Nagamatsu stated that the value of β was constant [Ref. 15] . However, Fig. 4 shows that β is greatly affected by the size of aggregate in concrete or by the size of specimen, and not constant. Hoshino and Tomeji showed that β was 5.25 for the direct tensile strength of mortar [Ref. 38] .

2) Effect of Size of Aggregates

Fig. 5 shows the relation between material constant (β) and width (diameter) (D) of spacimen to size of aggregate (d) ratio (D/d), in the tensile strength of concrete. In the case of PR-T specimen (prism specimen in direct tensile test), the value of β increases straight up to D/d=10, but after D/d exceeds 10, it shows tendency to decrease.

In the case of CY-SP specimen (cylinder splitting specimen), the value of β has the tendency to increase up to D/d=10 while showing considerable variabiability, but after D/d exceeds 10, it shows the tendency to decrease same as PR-T specimen.

(3) Coefficient of Variation of Strength

Coefficient of variation of strength (CV) was calculated by the following formula.

$$CV = \frac{1}{\bar{F}}\sqrt{\frac{\sum_{i=1}^{N}(Fi-\bar{F})^2}{N-1}} \quad \cdots\cdots\cdots\cdots(4)$$

where, CV : coefficient of variation

Fi : measured value of strength

$\bar{F}$: mean value of strength

N : total number of specimens

1) Effect of size of specimen

Fig. 6 shows the relation between coefficient of variation of strength (CV : %) and size of specimen (S). The value of CV of compressive strength of prism specimen decreases greatly with increase in prism width S in the range whear S is 4.46cm—7.25cm,but in the rang where S is larger than 7.25cm, the tendency of decrease suddenly becomes small. This tendency coincides well with the result of experiments reported by the auther [Ref. 7] .

The value of CV of compressive strength of cylinder specimen in the range where s is ϕ7.5-ϕ15 decrease continuously with increase in S. The value of CV of the direct tensile strength of prism specimen decreases greatly with increase in S in the rang where the prism width (S) is 4.46—7.25, but after the value of S exceeds 7.25 each series of concrete show very little decrease and show constant value. However, the specimen of which maximum size of aggregates is smaller, shows the smaller values of CV. On the other hand, the value of CV of the spling tensile strength of cylinder specimen decreases greatly with increase in S in the range where cylinder diameter (S) is 7.5cm —10cm, but after exceeding this rang the value of CV decreases very little and shows almost constant value. Hoshino reported that the variability of direct tensile strength was larger than the variablility of splitting tensile strength [Ref. 37] . However, according to this experimental results, the value of

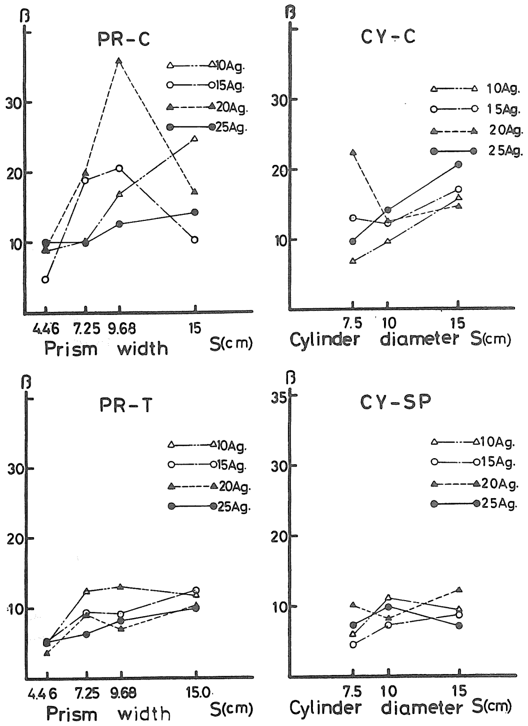

Fig. 4 Relation between material constant (β) and size of specimen S.

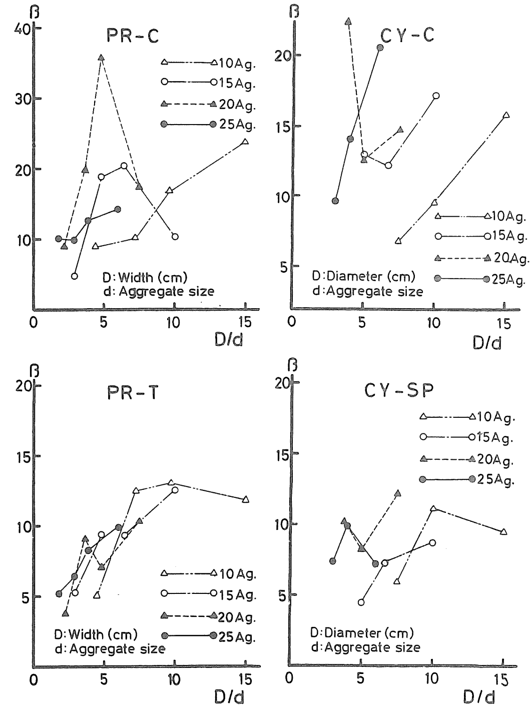

Fig. 5 Relation between material constant (β) and width (diameter) of specimen to size of aggregate d ratio D/d.

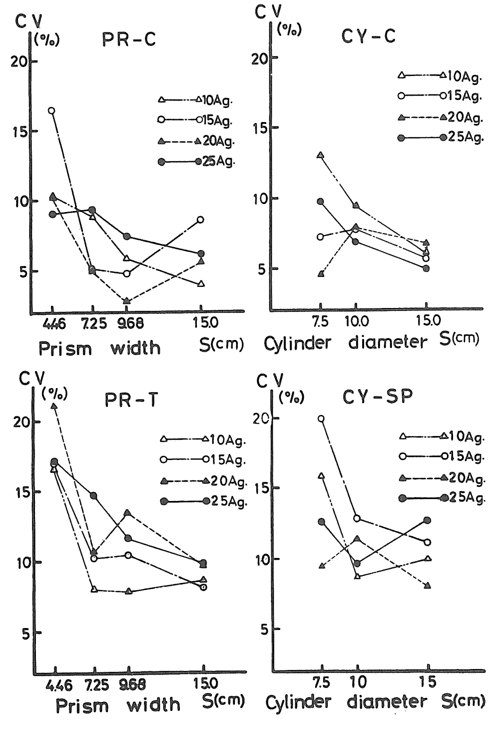

Fig. 6 Relation between coefficient of variation of strength (CV : %) and size of specimen (S).

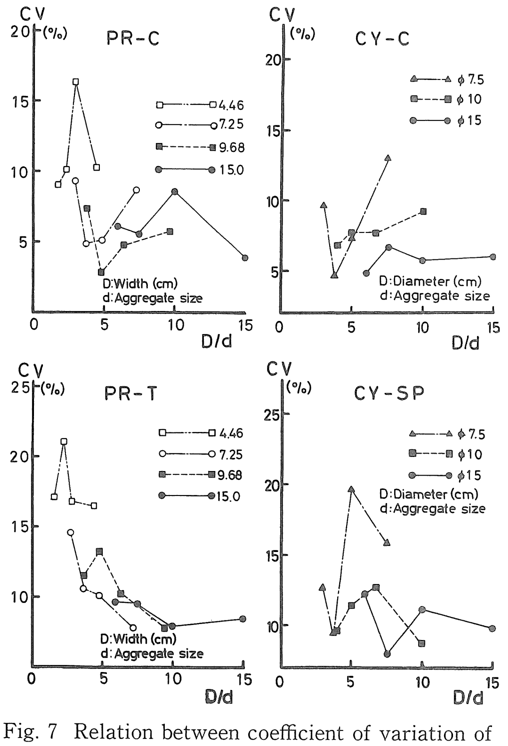

Fig. 7 Relation between coefficient of variation of strength (CV : %) and the value of D/d.

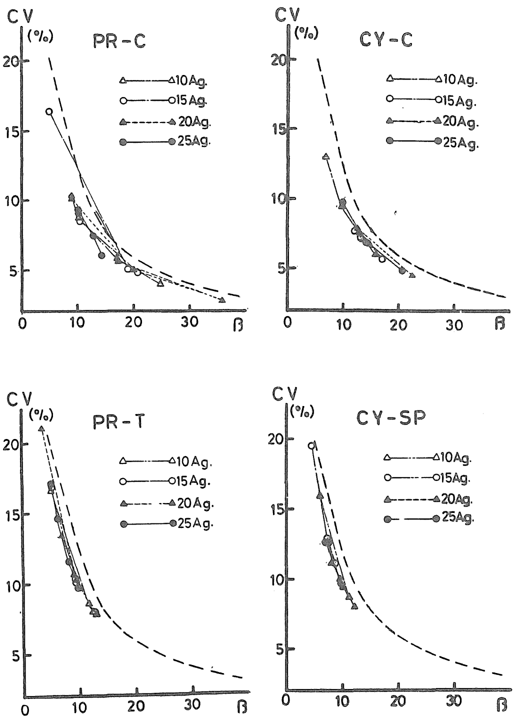

Fig. 8 Relation between coefficient of variation of strength (CV : %) and material constant (β).

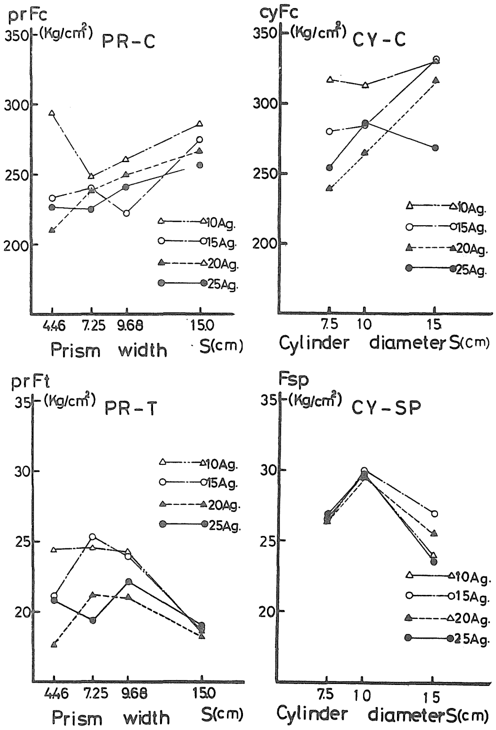

Fig. 9 Relation between strength (F) and size of specimen (S).

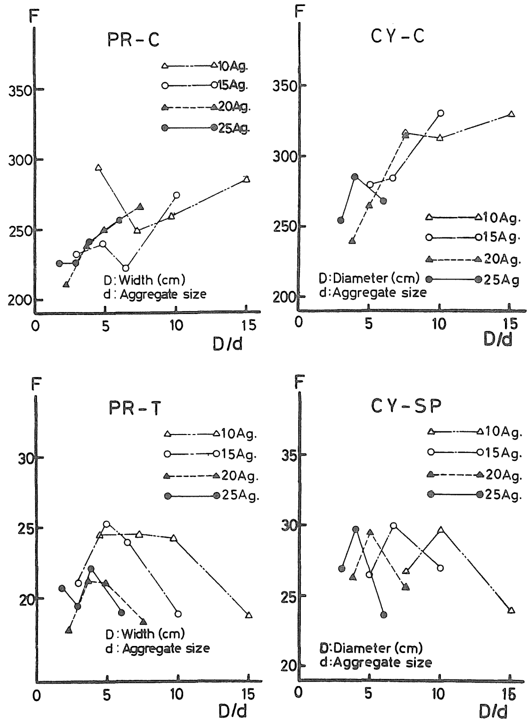

Fig. 10 Relation between strength (F) and the value of D/d.

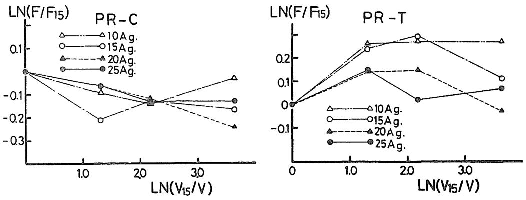

Fig. 11 Relation between relative concrete strength ($\ln F/F_{15}$) and relative volume of specimen ($\ln V_{15}/V$).

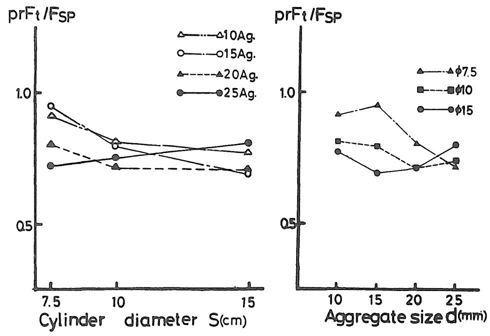

Fig. 12 Relation between prFt/Fsp and cylinder diameter S, and aggregate size d.

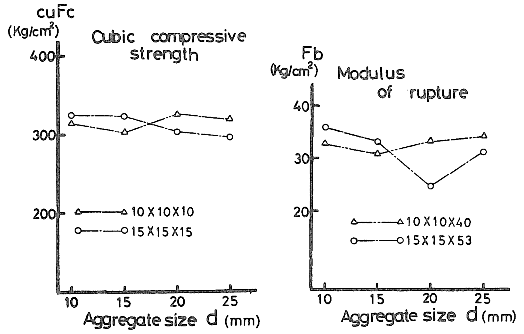

Fig. 13 Cubic compressive strength and modulus of rupture of prism specimen.

CV of direct tensile and splitting showed similar tendency and the variability of splitting strength showed slightly larger value.

2) Effect of Size of Aggregeate

Hoshino, Johnston and sidewell, Sabnis and Mirza, et al reported that the specimen with aggregate of large size showed larger variability of tensile strangth in the splitting tensile test [Ref. 37,43,18,46]. However, the experimental result of this time showed that specimens of 20 Ag. series and 25 Ag. series had smaller variablility than specinens of 15 Ag. series and 10 Ag. series. More researches are needed in this area. Johnston reported that the specimen with aggregate of large size showed larger value of CV of direct tensile strength in the direct tensile test, which concides with the experimental results of this time [Ref. 43].

Fig. 7 shows the relation between values of CV of direct tensile strength and D/d of prism specimen. The value of CV shows tendency to decrease with increase in D/d in the hyperbolic shape, but the relation between CV and D/d can not be indicated in one formula because it is affected by size of aggregate.

3) Effect of Value of β

Fig. 8 shows the relation between CV and β. The value of cofficient of variation (CV) can be calculated by the following theoretical equation [Ref. 15,21] .

$$CV = \sqrt{\frac{\Gamma(1+\frac{2}{\beta+1})}{\Gamma^2(1+\frac{1}{\beta+1})} -1} \quad \cdots\cdots\cdots(5)$$

where, Γ : gamma function

Nagamatsu reported that coefficient of variation CV of strength was affected only by material constant β, not affected by size of specimen and proporsed eq. 5. However, it is obvious in Fig. 8 that the experimental values are larger than the theoretical value in both prism compressive strength and prism direct tensile strength. Therefore, the probability distribution of its strength can not always be expected by a Weibull distribution [Ref. 22] .

(4) Mean Value of strength

Fig. 9 shows the relation between strangth (F) and size of specimens (S). Compressive strength of prism specimens decreases straight when size of specimen becomes smaller, and strangth decreases in parallel with increase in size of aggregate in concrete. These tendencies were already confirmed by the auther's previous study [Ref. 7] .

The direct tensile strength of prism specimen shows considerable increase with decrease in size of specimens S in the rang where S is 15.0—9.68cm, but this increase of direct tensile strength reaches the top when S is smaller than 9.68 and it rather decreases when S is smaller than 7.25.

These strengths of specimens of aggregate size 20 Ag. series and 25 Ag. series showed tendency to become lower than these strength of 10 Ag. and 15 Ag. series when the value of S become smaller. On the other hand, as shown in Fig. 9, splitting tensile strangth of cylinder specimen showed a little difference according to the size of aggregate in case of ϕ15cm cylinder-the specimen with aggregate of large size showing slightly low strength, but in case of ϕ10 and ϕ7.5cm cylinders, any difference of strangth by size of aggregate was not recognized. Besides, the value of splitting tensile strength increases greatly with decrease in the value of diameter S when S is 15—10cm, but with S=10cm as border,when S decreases from 10cm to 7.5cm strangth rather falls. On the other hand, Subnis and Mariza repored "Mirza [Ref. 46] tested series of cylinders, cast form the same model concrete, ranging from 1in. ×2in. (25mm×50mm) to 6in.×12in. (150mm×300mm) in splitting tensile tests, and the mean strength and the standard deviation were found to decrease with an increase in size of specimen [Ref. 20]. This difierence will be studied here after.

Fig. 10 shows the relation between strength (F) and the value of D/d. Compressive strength (F) of prism specimen and cylinder specimen shows tendency to increase with increase in D/d and in the case of same value of D/d, the larger the size of specimen is,the slightly larger value concrete shows.

On the other hand, with same D/d value, direct tensile strength of prism specimen showed that the specimens of 20 Ag. and 25 Ag. series had considerably lower value than the specimens of 10 Ag. and 15 Ag. series. The experiment of this time shows that tensile strangth of concrete is not determined only by D/d, but also affected greatly by size of aggregate.

Fig. 11 show the relation between relative concrete strangth (1n F/F_{15}) and relative volume of specimen (1n V_{15}/V) plotted on the logarithmic graph (both co-ordinates) where F_{15} and v_{15} are the strangth and volume of S=15.0cm series of prism specimen, respectively. According to the Weibull's weakest statistical theory and the stochastic theory for the perfectry brittle fracture mode, size effect of strength can be writtlen as follows:

$$\frac{F}{F_0}=\left(\frac{V_0}{V}\right)^{1/(\beta+1)} \quad \cdots\cdots\cdots\cdots\cdots\cdots(6)$$

where, F_0 and V_0 are strength and volum of standard specimen. β is material constant .

According to the above formula, relation between $\ln(F/F_0)$ and $\ln(V_0/V)$ is indicated on the straight line wlth the slope $-1/(\beta+1)$.

In Fig. 11, relation between compressive strength of prism and volume of specimen is shown nearly by the formula of straihit line regardless of aggregate size except secimen of S=4.46cm, and the above formula is almost affected, but in the case of direct tensile strength, it can not be considered as straight line. The value of direct tensile strength shows tendency to increase on the contrary when the value of specimen size becomes larger than the certain value.

Fig. 12. shows relationship between ${}_{pr}F_t/F_{sp}$ ratio (${}_{pr}F_t$=direct tensile strength of prism, F_{sp}=splitting tensile strength of cylinder) and size of aggregate. In Fig. 12, values of ${}_{pr}F_{t7.25}/F_{sp}\phi_{7.5}$, ${}_{pr}F_{t9.68}/F_{sp}\phi_{10}$ and ${}_{pr}F_{t15.0}/F_{sp}\phi_{15}$ were calculated, supporsing the size of prism S and diameter of cylinder ϕ were equal for convenience' sake. Fig. 13 shows the cubic compressive strength calculated from the mean value of each six specimens and the modulus of rupture calculated from the mean value of each three specimens, for referrence.

4. CONCLUSION

The following are the conclution of study on probability distribution and size effect, with tests of compressive strength, direct tensile strength and splitting tensile strength using different sizes of aggregate and specimen which are main factors to determine strength of concrete.

1) The experimental value of strangth shows the probability distribution quite close to the strainght line when plotted either on Weibull probability papers or on Normal ones, but some values are slightly apart from the straight line near the maximum and minimum experimental values.

2) The value of material constant β shows tendency to increase with increase in size of specimen S. The value of β is largely affected by the size of aggregate in concrete and by the size of specimen, and cannot be considered as constant value.

3) Coefficient of variation for strength in compressive test showed gradual increase with decrease of specimen size in the range of 15cm to 10—7cm, but it showed greatly increase when the specimen size becomes smaller than the range of 10—7cm.

4) The value of coefficient of variation (CV) for tensile strength showed same tendency as compressive strength, in both cases of direct tensile test and splitting test.

5) The value of coefficient of variation (CV) for tensile strength of concrete in direct tensile test shows larger values when the size of aggregate in the specimen is larger.

6) Experimental values of coefficient of variation (CV) of strength show the lower values than theoretical ones indicated by the formula (5). Therefore the probability distribution of its strength cannot always be expressed by the Weibull distribution.

7) Compressive strength decreases with decrease in size of specimen, both in prism and cylinder specimens, and with increase in size of aggregates.

8) Tesile strength of concrete reaches the top at specimen size of 10cm both in prism and cylinder specimens, but it rather decreases when specimen size becomes smaller than 10cm.

9) The formula (6) is almost effected in compressive test of prism specimen, but the formula (6) is not effected in direct tensile test.

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