Probabilistic Study of the Variability of Flexural Strength of Reinforced Concrete Simple Beam

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鉄筋コンクリート単純ばりの曲げ強度に関する確率論的研究

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This study, in the Experiment I and II, examined variability of strength of materials such as reinforcing bars and concrete composing reinforced concrete beams, and offered data to simulate occurrence of the probability distribution of materials used. In the Experiment III, fourty-eight reinforced concrete beams were fabricated, and the probability distridution of variations in steel placement, variability of the size of cross area were examined and variability of ultimate flexural strength of reinforced concrete simple beams were tested, Supposing the probability distribution of materials used and dimension variations obtained by the Experiment I, II and III as Normal distribution, the computer simulation was done by the Monte Carlo Method, and the result were compared with ones of ultimate flexural strength of reinforced concrete simple beams obtained by the Experiment III. By these comparative studies, possibility to presume the probability distribution of flexural strength of reinforeced concrete beams were examined.

1. Introduction

As strength of materials such as reinforcing bars and concrete composing reinforced concrete members is affected by various defects distributed at random within materials, the strength has variability peculiar to each material and should be treated as random variable. [Ref. 1,2,3,4] As to dimension variations of materials used and dimension variations at fabricating beams, the variability distribution of dimension is largely affected by the qurlity of construction works, and should be treated as random variable. [Ref. 5,6] Therefore, strength and deformation capacity of reinforced concrete members composed by these random variables are also probabilistic phenomena occuring from the combination of each random variable. These values should show the probability distribution and should be treated as random variable.

On the other hand, in case of calculating the probability distribution of strength of reinforced concrete members by Monte Carlo simulation by the computer, sumpling the probability distribution of strength of each material and dimension variations at fabricating test specimens has very important meaning, but data on these probability distributions are very scarce at present. [Ref. 2,5,6,7,8,9,10,11]

This study, in the Experiment I, examined variability of diameter, strength and elongation of four kinds of reinforcing bars, and in the Experiment II, examined variability of strength of concrete which composes reinforced concrete beams, and offered data to simulate occurence of the probability distribution of materials used. In the Experiment III, forty eight reinforced concrete beams with two kinds of similar dimensions using the reinforcing bars from the same lot and concrete from hte same batch tested in the Experiment I and II were fabricated, and the probability distribution of variations in steel placement and variability of the size of cross area were examined. Supposing the probability distribution of materials used and dimension variations obtained by the Experiment I,II and III as normal distribution, the computer simulation was done by the Monte Carlo Method, and the results were compared with ones of ultimate flexural strength of reinforced concrete simple beams obtained by the Experiment III. By these comparative studies, possibility to presume the probability distribution of flexural strength of reinforced concrete simple beams was examined.

2. Experiment I (Diameter, Elongation, Yield Strength of Reinforcing Bars)

The load that reinforcing bars can carry is determined not only by strength of bars but also by the area of cross section of bars. Generally, there is

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variability in strength of reinforcing bars and moreover, the coefficient of variance of the area of cross section becomes more than two times of the coefficient of variance of diameter. Therefore, variability of diameter of reinforcing bars largely affects the load carried by the reinforcing bars. By Aoki's study based on the test results of reinforcing bars for construction works examined at the Building Material Test Center, vareability of the area of cross section of bars is larger than expected in both of deformed bars and round bars, and it shows Log-normal distribution, and in case of strength, the form of distribution shows tendency to incline toword the smaller values of strength. [Ref. 2] On the other hand, from the results of mechanical properties of various reinforcing bars, Sher Ali Mirza,et al reports that variation in yield strength within a single bar is relatively small, while the in-batch variation for a given heat is slightly larger, and the variation of samples taken from different batches or sources is larger. [Ref. 7]

(1) Summary of Experiment

To examine the probability distribution of diameter, elongation, yield strength and ultimate tensile strength about reinforcing bars of ϕ6A, D10, D13 and D16, 150 bars each- total 600 bars were measured their area of cross section, length, weight and tension test was done. The dialgage type bar tension meter was used for measuring yield strength and the yield strength was confirmed by careful reading of scales on the tension meter, especially around yield strength.

(2) Test Results and Discussion

Table 2.1 shows a part of the results of tension test of reinforcing bars. M shows the average value, VA-variance, SD-standard deviation, and CD -coefficient of variance (%). The diameter of ϕ6A bars shows coefficient of variance of 1.47%, but the values of D10, D13 and D16 bars are smaller as 0.43% -0.80%. It is considered because ϕ6A bars, not for structural use, are not ristricted severely regarding the area of cross section. These values, except ϕ6A, show the smaller values cempared with ones of Sher Ali Mirza, et al [Ref. 7] and lower than Aoki's value (0.9%-1.45%) [Ref. 2] that is the test result of the Test Center. Among these reinforcing bars, ϕ6A and D10 bars and their data were used for the beam test (reported later) and for simulation. Yield strength and tensile strength shew the same tendency and sher Ali Mirza, et al showed that the coefficint of variation was, in general, in the order of 1%-4% for individual bar sizes and 5%-8% for individual bar sizes when data were taken from many sources. [Ref. 7] On the other hand, Aoki's value shows 4.5%-8.4% for yield strength and 4.3%-8.5% for tensile strength, and these two values are similar. [Ref. 2] The test results of this time, except ϕ6A, show 1.1%-1.6% for yield strength and 0.64%-2.63% for tensile strength, and similar to the deta by sher Ali Milza, et al. Julian reports the coefficient of variance of 12% for 40 ksi steel. [Ref. 10] Horikawa shows that variability of elongations larger compared with variability of strength, and the result of this experiment also shows large values as 5.1%-6.2%. [Ref. 12] Fig. 2.1 shows yield strength of reinforcing bars by Weibull distribution, by Normal distribution and by Log-normal distribution. The straight lines in the figures were

Table 2.1 Test result of reinforcing bars.

		diameter ϕs	yield str. σ_{sy}	ultimate str. σ_{su}	elong-ation
$\phi 6_A$	M	5.65 mm	4020kg/cm²	4750kg/cm²	27.4%
	VA	0.00689	15900	20200	6.13
	SD	0.0830mm	126 kg/cm²	142 kg/cm²	2.48%
	CV	1.47 %	3.14 %	2.99 %	9.05%
D10	M	9.18	4140	6020	23.6
	VA	0.00537	4400	24900	2.15
	SD	0.0733	66.4	158	1.47
	CV	0.799	1.60	2.63	6.23
D13	M	12.25	3779	5515	24.9
	VA	0.0039	1627	1237	1.66
	SD	0.0627	40.34	35	1.29
	CV	0.51	1.07	0.64	5.17
D16	M	15.8	4010	6140	23.6
	VA	0.00456	3260	8060	1.48
	SD	0.0675	57.1	89.8	1.22
	CV	0.428	1.42	1.46	5.13

Table 2.2 Correlation coefficient for the straight line calculated by the method of least square of experiment values of yield strength.

	Weible dist.	Log-normal dist.	Normal dist.
$\phi 6_A$	0.976	−0.995	−0.999
D10	0.992	−0.992	−0.993
D13	0.980	−0.965	−0.996
D16	0.951	−0.985	−0.986

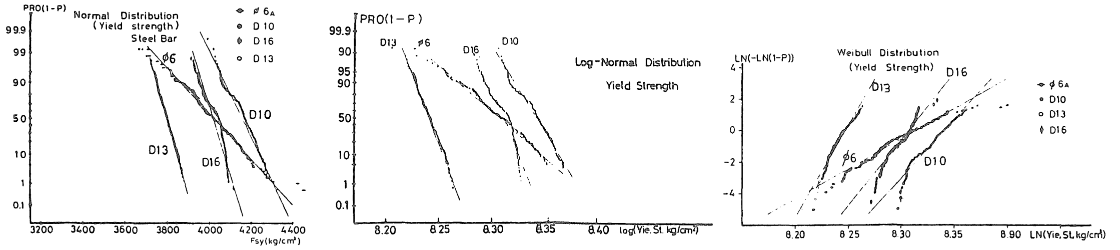


Fig. 2.1 Probability distribution for yield strength of reinforcing bars.

obtained by the method of least squares, and Table 2. 2 shows correlation coefficient for the straight line formula of experiment values. Every correlation coefficient shows more than 0.98 and they are close to the straight line and can be indicated well by both of Normal distribution and Log-normal distribution, but in Weibull distribution, both ends of distribution separate slightly from the straight line. On the other hand, by Nishimura's report which examined mechanical properties of steel, [Ref. 9] both of yield strength and tensile strength of steel show Log-normal distribution, and Freudenthal confirmed that yield strength of ASTM A7 steel by Mill Test was coincident with Log-normal distribution. [Ref. 11] Sher Ali Mirza, et al [Ref. 7], considering that normal distribution or log-normal distribution does not coincide for yield strength at both ends of data, proporsed the new equation of probability density function. The test of this time is the result of one lot, and quite coincident with both of Normal distribution and Log-normal distribution.

3. Experiment II (Experiment on compressive and Splitting Tensile Strength of Concrete)

(1) Summary of Experiment

Table 3.3 shows the actual dimensions of three kinds of concrete prism specimens fabricated for the experiment. 15 prisoms each of three kinds and total of 45 prisms were fabricated, cast by concrete of the same mix proportion, and 100 concrete cylinders of ϕ10x20cm were made at the same time in each casting for examination of the probability distribution of compressive strength and splitting tensile strength. Both ends of concrete prism specimens had steel mold and concrete was cast sideways.

Common Portland cement and, as aggregates, Yahagi River sand less than 5mm and Tenryu River gravel less than 15mm were used. The water cement ratio of concrete was 60% and Table 3.1 shows its mix proportion. Concrete was mixed by the Smith type concrete mixer with the maximum capacity of 600 1 and casting was done at the same time with the beam specimens (reported later). Both of concrete prisms and cylinders were remolded after two days and examined after seven weeks.

(2) Test Results and Discussion

Table 3.2 and Fig. 3.1 show the test results of compressive and splitting tensile strength of ϕ10x20 cm concrete cylinders and their probability distribution. In the mark CP＊2-AC, CP means cylinder, no.2 means the first casting, AC means thecompressive test by the Amsler Type hydraulic compressive machine. (2-4) shows the result considering the total of 2-4 as one population. Essentially, data should be treated for three different populations, but three populations had very close properties and therefore, is indicated as one populaton. AT shows the test results of splitting tensile strength. r shows correlation coefficient, μ in Normal distribution shows the medium value of the Normal distribution curve supposing the test results as Normal distribution. σ

Table 3.1 Concrete mix proportion.

Concret mix	Water (kg/m³)	Concret (kg/m³)	Sand (kg/m³)	Gravel (kg/m³)	s/a (%)	design slump (cm)
	215	358	810	887	49	15

Table 3.2 Test results of compressive strength and splitting tensile strength of ϕ10x20cm concrete cylinder.

		No. of specimen	Mean kg/cm²	SD kg/cm²	VA	CV %	Weible dist. appro. eq.	Weible dist. r	Normal dist. μ	Normal dist. σ	Normal dist. r′
cylinder compressive strength	CP＊2—AC	40	260	14.8	219	5.68	19.9X—111	0.976	261	16.4	—0.988
	CP＊3—AC	34	264	14.9	221	5.64	19.6X—110	0.958	264	16.7	—0.985
	CP＊4—AC	31	260	14.0	196	5.39	20.6X—115	0.995	260	15.8	—0.981
	CP＊(2～4) AC	105	261	14.6	212	5.57	21.4X—120	0.983	262	15.5	—0.997
splitting strength	CP＊(2～4) AT	101	25.8	28.2	7.94	10.9					

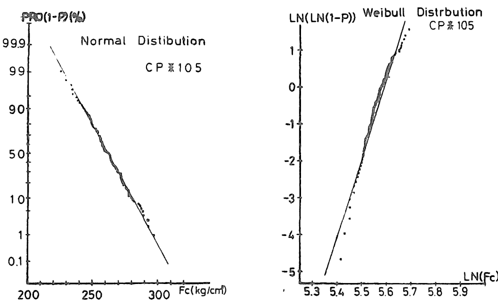


Fig. 3.1 Probability distribution of compressive strength for ϕ10x20 cm concrete cylinder (CP＊(2～4) AC).

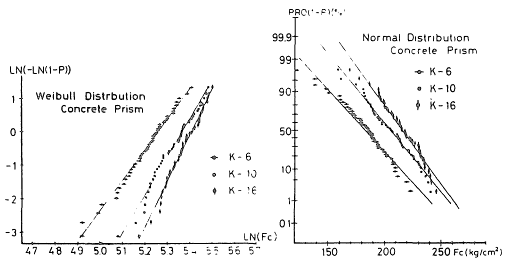


Fig. 3.2 Probability distribution of compressive strength for concrete prisms (KP＊(2～4) AC).

Table 3.3 Test results of compressive strength of concrete prisms.

		No. of speci.	b (cm)	h (cm)	l (cm)	Prism compressive strength			Weibull dist.		Normal dist.		
						M	SD	CV	LN(-LN(1-p))	r	μ	σ	r′
KP＊2	−6	15	4.46	4.66	13.4	180	17.7	9.82	10.4X−54.6	0.959	180	21.4	−0.967
	−10	15	7.22	7.36	21.8	198	18.6	9.40	10.8X−57.3	0.967	198	22.5	−0.967
	−16	15	12.4	12.6	37.4	212	10.7	5.03	20.5X−11.0	0.979	212	12.6	−0.990
KP＊3	−6	15	4.46	4.56	13.4	190	20.8	12.3	6.48X−34.5	0.879	193	32.1	−0.867
	−10	15	7.22	7.37	21.8	209	20.8	9.91	9.95X−53.6	0.980	210	25.1	−0.967
	−16	15	12.4	12.5	37.4	209	20.7	9.91	10.2X−55.2	0.993	209	24.7	−0.981
KP＊4	−6	15	4.46	4.58	13.4	174	21.5	12.4	8.15X−42.5	0.988	174	25.5	−0.986
	−10	15	7.22	7.40	21.8	207	20.5	9.89	10.5X−56.2	0.986	207	24.1	−0.994
	−16	15	12.4	12.6	37.4	226	16.3	7.23	13.9X−75.6	0.979	226	19.9	−0.960
KP＊ 2~4	−6	45	4.46	4.60	13.4	182	22.2	12.2	9.00X−47.3	0.995	183	24.5	−0.987
	−10	45	7.22	7.39	21.8	205	20.1	9.84	11.5X−61.6	0.989	205	22.1	−0.993
	−16	45	12.4	12.6	37.4	216	17.6	8.18	13.8X−74.6	0.995	216	19.4	−0.989

shows standard deviation, r' shows correlation coefficient for the approximate straight line of data in Normal distribution. The result of this time is closer to Normal distribution than Weibull distribution. By Cook's report, cofficient of variance of concrete with compressive strength of 3.46 ksi shows 12%. [Ref. 8] On the other hand, by bruce's report, coefficient of variance of concrete with compressive strength of 3.5 ksi is 12%. [Ref. 6] Besides, data of concrete of used by Robert, et al for Monte Carlo simulation was the one in Normal distribution that showed coefficient of variance of 17.6%. The result of this time shows coefficient of variance of 5.39%-5.68%, much smaller than values in the papers refered.

Table 3.3 and Fig. 3.2 show the test results of compressive strength for concrete prisms with three kinds of similar dimensions and their probability distribution. The coefficient of variance for compressive strength of concrete prisms shows 5.03% -12.3% and shows slightly larger variability than ones of concrete cylinder . It is contidered because wooden molds were used for sides of specimens at fabricating them, and the height/width(diameter) ratio of concrete prisms is 3:1 while it is 2:1 in concrete cylinders. Generally, smaller specimens have larger coefficient of variance compared with larger specimens, and specimens should be fabricated carefully. Their distributions, except KP＊3-6, are quite close to either of weibull distribution or Normal distribution.

4. Experiment III (Flexural Test of Reinforced Concrete Simple Beams)

Total of 48 reinforced concrete beams with two kinds of dimensions were fabricated using the same reinforcing bars and concrete examined in the Experiments I & II, under three separate operations. Flexural tests were done for simple beams and the probability distribution of the ultimate flexural strength was examined.

(1) Summary of Experiment

Fig. 4.1 and Table 4.1 show details of beam specimen. Stirrups were placed densely to prevent shear failure. BA-6 beams and BA-10 beams were fabricated to have similar dimensions of cross section, main reinforcement ratio and stirrup ratio. Table 4.1 shows the objective dimensions for fabrication. Beams were tested by two-point loading condition having three same spans. Load was measured by the 3-ton and 5-ton load-cell, and deflection was measured by the slide type differential transformer with digital strain meter.

(2) Test Results and Discussion

Table 4.2 shows the actual dimensions of beam specimen after removing mold. As they were carefully fabricated in the laboratory to have precise dimensions, the coefficient of variance is quite small, but this preciseness is different from the one at the construction site. These values were measured for simulation.

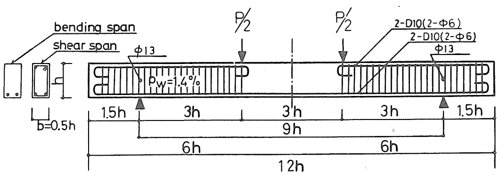
Table 4.3 and 4.2 show the test results of ultimate flexural strength and their probability distribution. Fig. 4.2 shows results of conversion from the beam with two $\sigma_{sy}=4000$ kg/cm^2 bars as main reinforcement at top and bottom of the beam to the one with dimension of 2.789 times of BA-6 beam. The coefficient of variance for the ultimate flexural strength on beam specimens is 2.1%-3.3% and qute small compared with variability of yield strength of steel bars.

5. Monte Carlo Simulation of the Ultimate Flexural Strength of Reinforced Concrete Simple Beams

The ultimate flexural strength of reinforced concrete simple beam is affected by the variations in the strength of concrete and reinforcement, the cross section dimension and steel placement.

The effects of these variables on the variability of flexural strength were studies using the Monte Carlo simulation.

The fundamental ploblem of sampling in the Monte Carlo simulation is to choose the suitable values of the probability distribution among the already known random variables as values of the probability distribution of mechanical properties of each material. On solution of this problem, occurence of random variables can simulated by the physical probabilistic process.



BEAM BA-10(BA-6)

Fig. 4. 1 Outline of beam specimen BA-10 (BA-6).

The pseudorandom numbers are generated by D. H.Lemer's multiplicative cargruential method for the computer simulation in this study. [Ref. 13,14] The procedure for generating pseudorandom numbers are shown below.

$$n_{i+1} = 7^5 \text{ x } n_i$$

where, n_i is the integer number of eight figures. The upper bit of n_{i+1} is rejected by overflow in the computer and remained lower bit n_{i+1} is smaller than$2^{31}-1$.

$$r_{i+1} = \frac{n_{i+1}}{2^{31}-1}$$

Table 4. 1 Details of beam specimen.

beam series	Main reinforcement			Cross section			Shear span	Bending span	Span	Beam length	Stirrup		Scale ratio
		diameter (mm)	cross sec. area (cm²)	b (cm)	h (cm)	d (cm)	3h (cm)	3h (cm)	9h (cm)	12h (cm)		spacing (cm)	
BA- 6	2-φ 6	5.65	0.249	4.47	8.93	8.04	26.79	26.79	80.37	107.16	φ2.6	1.59	0.616
BA-10	2-D10	9.16	0.662	7.25	14.49	13.04	43.47	43.47	130.41	173.88	φ4.5	2.93	1

Table 4. 2 Actual dimensions of beam specimen after removing mold.

	Beam width			Normal distribution			Beam depth			Normal distribution			Effective depth			Normal distribution		
	M	SD	CV	μ	σ	r	M	SD	CV	μ	σ	r	M	SD	CV	μ	σ	r
BA∗ 6 - 2	4.49	0.021	0.47	4.49	0.025	−0.97	9.00	0.045	0.50	9.00	0.056	−0.98	8.15	0.114	1.40	8.15	0.180	−0.79
BA∗ 6 - 3	4.50	0.016	0.35	4.50	0.018	−0.99	9.03	0.032	0.35	9.03	0.039	−0.97	8.23	0.229	2.78	8.24	0.374	−0.77
BA∗ 6 - 4	4.55	0.026	0.57	4.55	0.032	−0.97	9.11	0.113	1.24	9.11	0.144	−0.98	8.16	0.052	0.64	8.16	0.067	−0.97
BA∗10- 2	7.35	0.082	1.11	7.35	0.126	−0.81	14.67	0.086	0.59	14.67	0.109	−0.98	13.03	0.077	0.59	13.02	0.102	−0.94
BA∗10- 3	7.35	0.137	1.86	7.35	0.260	−0.66	14.62	0.065	0.44	14.62	0.082	−0.95	12.99	0.062	0.47	13.00	0.082	−0.93
BA∗10- 4	7.33	0.026	0.36	7.30	0.033	−0.99	14.67	0.068	0.46	14.67	0.086	−0.97	12.99	0.108	0.83	12.99	0.141	−0.96
BA∗ 6 -2∼4	4.51	0.033	0.72	4.51	0.038	−0.98	9.05	0.084	0.93	9.05	0.103	−0.91	8.18	0.148	1.81	8.19	0.218	−0.77
BA∗10-2∼4	7.33	0.092	1.26	7.34	0.146	−0.71	14.05	0.080	0.55	14.66	0.084	−0.98	13.00	0.088	0.68	13.00	0.096	−0.96

Table 4. 3 Test results of ultimate flexural strength of beam specimen.

	No. of speci.	Beam ultimate flexural strength (P)					
		M (ton)	SD (ton)	CV (%)	Normal distribution		
					μ	σ	r
BA∗ 2 — 6	8	1.162	0.029	2.49	1.16	0.04	−0.94
BA∗ 3 — 6	8	1.148	0.027	2.32	1.15	0.03	−0.98
BA∗ 4 — 6	8	1.096	0.027	2.47	1.10	0.04	−0.95
BA∗ 2 —10	8	3.060	0.101	3.31	3.06	0.15	−0.85
BA∗ 3 —10	8	3.086	0.065	2.10	3.09	0.08	−0.96
BA∗ 4 —10	8	3.074	0.086	2.79	3.07	0.11	−0.98
BA∗(2∼4)- 6	24	1.135	0.039	3.44	1.14	0.05	−0.98
BA∗(2∼4)-10	24	3.073	0 082	2.68	3.07	0.10	−0.98

where, r_{i+1} is uniformly distributed pseudo-random number generated between O and l. Fig. 5.1 shows the flow chart of generating of random numbers. Essentially, 2^{35}-1 should be used, but $2^{31}-1$=2,147,483,647 were used because of convenience of the computer. Because the probability distributions of the Experiment I, II and III showed the result closer to Normal distribution than to other probability distribution, the Normal distribution type random numbers were generated by Direct Method (Inverse Transformation Method) using the uniformly distributed pseudorandom numbers generated by multiplicative

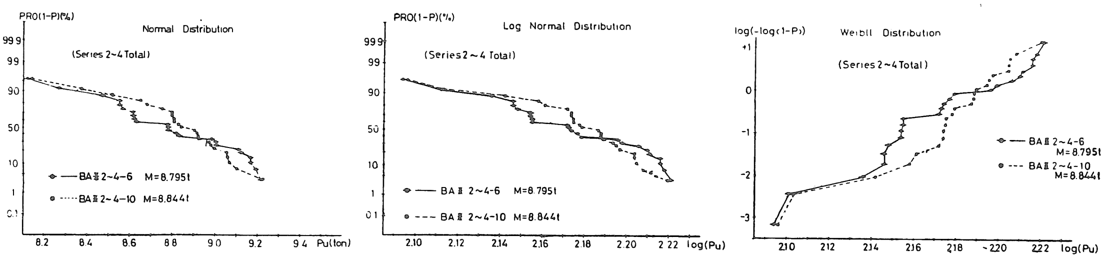


Fig. 4. 2 Probability distribution of ultimate flexural strength of beam specimen.

cargruential method.

The random variable x of probability density function f(x) was divided into eighty divisions between M-4σ(σ: standard deviation, M: mean value) and M+4σ, and the area of each divisions were integrated, and the values of existence probabilities at each eighty points were calculated. The Normal distribution type randm numbers were generated using the relationship between the pseudorandom number and the discrete cumulative frequency function derived from the value of existence probabilities.

The random variables used in this simulation are beam width b, effective depth d, bar diameter ϕ_s, bar yield strength σ_{sy}, young's modulus of concrete Ec, young's modulus ratio n, compressive strength of

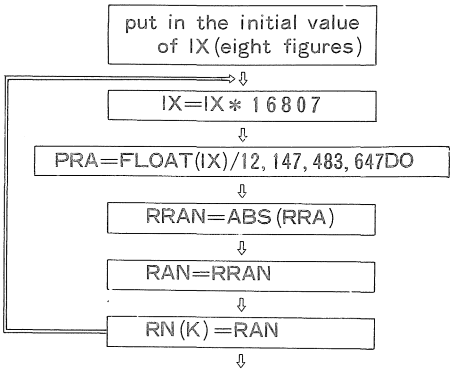


Fig. 5. 1 Flow chart of generating of random number using Lehmer's multiplicative corgruential method.

Table 5. 1 Input data for the beam Monte Carlo simulation and properties of pseudorandom number.

	BA＊(2～4)－6						BA＊(2～4)－10					
	Input data			Pseudorandom number			Input data			Pseudorandom number		
	M	SD	CV(%)	M	SD	CV(%)	M	SD	CV(%)	M	SD	CV(%)
b cm	4.55	0.023	0.51	4.55	0.040	0.88	7.46	0.066	0.88	7.46	0.130	1.74
b cm	8.18	0.041	0.50	1.18	0.119	1.45	13.0	0.012	0.092	13.00	0.098	0.76
ϕ_s mm	5.65	0.083	1.47	5.65	0.112	1.97	9.18	0.073	0.80	9.18	0.139	1.51
σ_{sy} kg/cm²	4020	126	3.14	4032	133	3.29	4140	66.4	1.60	4144	76.0	1.83
E_c ($\times10^5$) kg/cm²	1.78	0.34	19.1	1.80	0.35	19.3	1.78	0.34	19.1	1.80	0.35	19.3
n	11.80	0.34	19.1	12.14	2.73	22.5	11.80	0.34	19.1	12.14	2.73	22.5
a cm	26.8						43.5					
Es ($\times10^6$) kg/cm²	2.10						2.10					
Prism : $_pF_c$ kg/cm²	182	21.8	12.2	180	21.5	12.0	205	20.6	10.1	204	20.4	10.0
Cylinder : $_cF_c$ kg/cm²	261	14.6	5.57	262	14.5	5.54	261	14.6	5.57	262	14.5	5.55

Table 5. 2 Comparison between the test results of ultimate flexural strength of beam and the ones by the Monte Carlo simulation.

			No. of speci.	M (ton)	SD (ton)	CV (%)	normal dist.		
							μ	σ	r
BA＊(2～4)－6	test result		24	1.135	0.039	3.44	1.14	0.05	0.98
	simulation	prism	1000	1.011	0.043	4.26			
		cylinder	1000	1.081	0.041	3.80			
BA＊(2～4)－10	test result		24	3.073	0.082	2.68	3.07	0.10	0.95
	simulation	prism	1000	2.734	0.078	2.87			
		cylinder	1000	2.853	0.067	2.37			

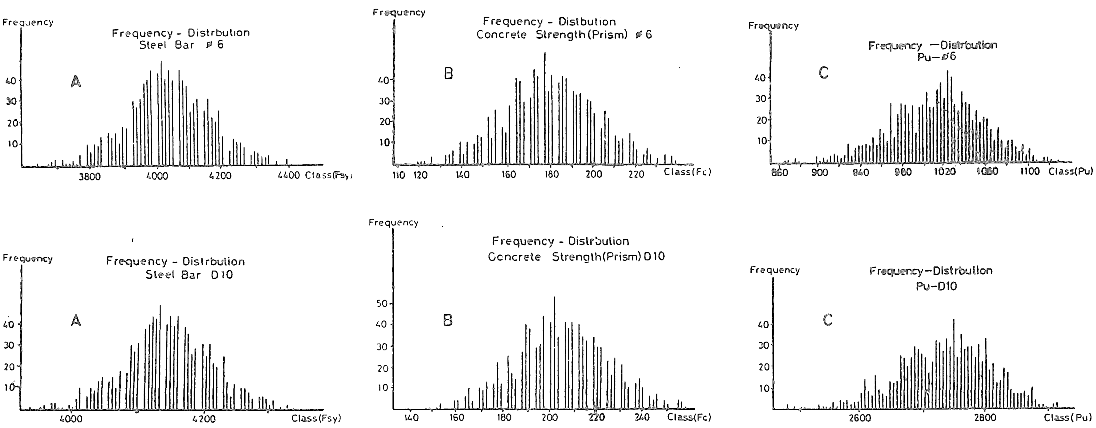


Fig. 5. 2 Frequency distribution histgram of random numbers of yield strength of steel bars and compressive strength of concrete prisms, and ultimate flexural strength of beam model calculated by Monte Carlo simulation.

concrete prism ${}_{p}F_{c}$ and compressive strength of concrete cylinder ${}_{c}F_{c}$. The Normal distribution type random numbers were generated for each parameter one by one to calculate the beam ultimate flexural strength.

Table 5.1 shows the result of statistical treatment of a thousand of the Normal distribution type random numbers generated using the results of the Experiments I, II and III as input data, supposing them as Normal distribution. Generally, the coefficiet of variance for random numbers show the slightly large values. It is because the section to occur the normal distribution type random numbers was divided into eighty divisions and the discrete cumulative frequency function was derived, and preciseness inproves by increasing number of divisions.

The assumption to calculate the ultimate flexural strength are shown below.

i) In every portion of beams, the values of the mechanichal properties of steel bars and concrete are just the same as the random number generated by computer.

ii) In the center bending span of beams, the dimensions of beams are just the same as the random numbers generated by computer.

iii) The stress-strain curve of concrete is bi-linear with compressive strength 0.85 F_c and ultimate strain 0.003.

iv) The stress-strain curve of steel bars are bi-linear with young's modulus $2.1 \times 10^6 kg/cm^2$. In each calculations, each Normal distribution type random numbers generated by computer are assumed as the decision variate in the beam and simulated to calculate the beam ultimate flexurl strength.

Table 5.2 shows comparison between the test results of ultimate flexural strength of beams and ones by the Monte Carlo simulation. With either of data of concrete prisms or concrete cylinders as the value of concrete strength, the simulated results of coefficient of variance of ultimate flexural strength stowed similar values to ones by experiments.

Feg. 5.2 shows the frequency distribution histgram of random numbers of yield strength of steel bars and compressive strength of concrete prisms by simulation, and the histogram of ultimate flexural strength of beams. It is interesting that the histogram of ultimate flexural strength (results of simulation) show the form of distribution inclining slightly toward right, while the histogram of numbers, being normal distribution, is symmetrical on both sides of the mean values.

6. Conclusion

The following conclusion were obtained as the result of the Monte Carlo simulation using values of the probability distribution obtained by the experiments to examine the probability distribution of concrete strength and steel bars which are the main parameters to determine the strength of reinforced concrete beams, and the probability distribution of cross section dimentions of beams and placement variation of reinforcing bars, as input data.

(1) In case of specimens with reinforcing bars from the same lot, as in this experiment, both yield strength and tensile strength show the quite small coefficient of variance, but the coefficiet of variance of elongation showed large values. In this test, both yield strength and tensile strength showed distribution which was quite coincident with either Normal distribution or Log-normal distribution.

(2) The values of compressive strength of concrete cylinders showed the result closer to Normal distribution than to Weibull distribution. The values of coefficiet of variance of concrete strength showed much smaller value than ones used by Cook or Robert, et al.

(3) The value of compressive strength of concrete prisms showed much larger valiability than one of concrete cylinders, but its distribution was quite close either to Weibull distribution or Normal distribution.

(4) The value of coefficient of variance of flexural strength of beam specimens was 2.1%-3.3% and quite small compared with variability of concrete strength.

(5) Pseudorandom numbers occured by simulation of this time showed the tendency to have small variations when small values of coefficient of variance were input.

(6) The values of coefficient of variance of ultimate flexural strength of beams obtained by the Monte Carlo simulation was quite close to the experimental results, and it showed possibility to presume the probability distribution of flexural strength of beams by simulation.

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