On the regularity of function-kernels and the behavior of potentials in a neighborhood of the point at infinity

Dedicated to Professor Mitsuru NAKAI on the occasion of his 60th birthday

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関数核の正則性とポテンシャルの無限遠点の近傍での挙動について

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Abstract.

Let G be a symmetric and continuous function-kernels on a locally compact Hausdorff space X and δ be the point at infinity.

In this paper, first we define the several notions of thinness of a closed set at infinity δ and investigate the mutual relations among them.

For a non-negative G-superharmonic function u, we denote by $R_G^{X,\delta}(u)$ the G-reduced function of u to δ. A kernel G satisfying the domination principle is said to be regular when we have $R_G^{X,\delta}(G\mu)(x)=0$ G-nearly everywhere on X for the potential $G\mu$ of any positive mesure μ with compact support.

The regularity of kernels plays an important role in the theory of Hunt kernels.

The purpose of this paper is to characterize the regularity of function-kernels by the behavior of potentials in a neighborhood of infinity δ.

We shall prove that a continuous function-kernel G is regular if and only if, for every mesure μ with finite G-energy, the potential $G\mu$ is equal to 0 G-quasi-everywhere at infinity δ under the assumption that G satisfies the complete maximum principle.

1. Preliminaries

Let X be a locally compact Hausdorff space with countable basis. A non-negative function $G=G(x, y)$ on $X\times X$ is called a continuous function-kernel on X if $G(x, y)$ is continuous in the extended sense on $X\times X$, finite except for the diagonal set of $X\times X$ and $0<G(x, x)\leqq+\infty$ for any $x\in X$. The kernel $\check{G}$ defined by $\check{G}(x, y)=G(y, x)$ is called the adjoint kernel of G.

We denote by M the set of all positive measures on X. The potential $G\mu(x)$ and the adjoint potential $\check{G}\mu(x)$ of $\mu\in M$ are defined by

$$G\mu(x)=\int G(x, y)d\mu(x) \quad \text{and} \quad \check{G}\mu(x)=\int \check{G}\mu(x)d\mu(x)$$

respectively.

The G-energy of $\|\mu\|$ of $\mu\in M$ is defines by $\|\mu\|^2=\int G\mu(x)d\mu(x)$.

Put

$$M_o=\{\mu\in M \ ; \text{ support } S\mu \text{ of } \mu \text{ is compact}\},$$

$E_o(G)=\{\mu\in M_o\ ;\ \|\mu\|<+\infty\}$,

$F_c(G)=\{\mu\in E_o(G)\ ;G\mu(x)$ is finite and continuous on $X\}$.

A Borel measurable set B is said to be G-negligible if $\mu(B)=0$ for every $\mu\in E_o(G)$. We say that a property holds G-nearly everywhere on a subset A of X (written symply G-$n.e.$ on A), when it holds on A except for a G-negligible set.

A non-negative lower semi-contionuous function $u(x)<+\infty$ G-$n.e.$ on X is said to be *G-superharmonic* when for any $\mu\in E_o(G)$, the inequality $G\mu(x)\leqq u(x)$ G-$n.e.$ on $S\mu$ implies the same inequality on the whole space X.

We denote by $S(G)$ the totality of G-superharmonic functions on X and by $P_{M_O}(G)$(resp. $P_{E_O}(G)$) the totality of G-potentials of mesures in $M_o(G)$(resp. $E_0(G)$).

The potential throretic principles are stated as follows.

(I) We say that G satisfies the *domination principle* and write symply $G\prec G$ when $P_{M_O}(G)\subset S(G)$.

(II) We say that G satisfies the *maximum principle* when $1\in S(G)$

(III) We say that G satisfies the *complete maximum principle* when, for any non-negative number a, $P_{M_0}(G)\cup\{a\}\subset S(G)$.

(IV) We say that G satisfies the *balayage principle* if, for any compact set K and any $\mu\in M_o$, there exists a measure μ' *in* M_o, called a *balayaged measure* of μ on K, supported by K satisfying

$G\mu'(x)=G\mu(x)$ G-$n.e.$ on K,

$G\mu'(x)\leqq G\mu(x)$ on X.

(V) We say that G satisfies the *equilibrium principle* if, for any compact set K, there exists a measure γ in M_o, called an *equilibrium measure* of K, supported by K satisfying

$G\gamma(x)=1$ G-$n.e.$ on K,

$G\gamma(x)\leqq 1$ on X.

(VI) We say that G satisfies the *continuity principle* if, for $\mu\in M_o$, the finite continuity of the restriction of $G\mu(x)$ to $S\mu$ implies the finite continuity of $G\mu(x)$ on the whole space X.

REMARK 1. On the relations between the potential theoretical principles, the following results are well known.

(1) If a continuous functiong-kernel G satisfies the domination principle, then both G and $\check{G}$ satisfiy the continuity principle(cf. [6]).

(2) For a continuous function-kernel G, the following four statements are equivalent (cf. [6] and [9]) :

(a) G satisfies the domination principle.
(b) $\check{G}$ satisfies the domination principle.
(c) G satisfies the balayage principle.
(d) $\check{G}$ satisfies the balayage principle.

(3) For a continuous function-kernel G, the following statements are equivalent (cf. [3]and[9]) :

(a)G satisfies the maximum principle.

(b)$\check{G}$ satisfies the positive mass principle, namely, for $\mu\in E_o(G)$and $\nu\in M_o$, an inequality $\check{G}\mu(x)\leqq \check{G}\nu(x)$ on $S\mu$ implies the inequality $\int d\mu\leqq \int d\nu$.

If we suppose further that $\check{G}$ satisfies the continuity principle, then (a) and (b) are equivalent to (c) :

(c)G satisfies the equilibrium principle.

(4) Let G be a continuous function-kernel on X. Then G satisfies the complete maximum principle if and only if G satisfies both the maximum principle and the domination principle (cf. [3] and[9]).

(5) Suppose that $\check{G}$ satisfies the continuity principle. Then a property holds *G-n.e. on a subset A of X*, if and only if it holds *μ-a.e. on* A for every $\mu\in F_C(\check{G})$ such that $S\mu\subset A$.

2. Thinness of a closed set at infinity

In this section, we define the several notions of thinness of a closed set at infinity δ and compare these notions. We shall obtain the mutual relations holding among them.

For any compact K and any set A in X, the *G-capacity* $cap_G(K)$ of K and the *inner G-capacity* $capi_G^i(A)$ of A are defined respectively by

$$cap_G(K)=inf\{\int d\mu\ ;\mu\in M,\ G\mu(x)\geqq 1\ G\text{-n. e. on } K \text{ and } S\mu\subset K\},$$

$$cap_G^i(A)=sup\{cap_G(K);K \text{ is compact contained in } A\}.$$

DEFINITION 1. We say that a subset A of X is *thin at infinity δ in the sense of G-capacity* (written symply *G-cap. thin at δ*) when we have

$$\inf_{\omega\in\Omega_O} cap_G^i(A\cap C\omega)=0$$

where Ω_O denotes the totality of all relatively compact open sets in X.

For a Borel function u and a closed set F, we put

$$S_u^F(G)=\{\ v\in S(G)\ ;v(x)\ \geqq\ u(x)\ G\text{-n.e. on } F\}.$$

The *G-reduced function of u on F* and the *G-reduced function of u at infinity δ on F* are defined respectively by

$$R_G^F(u)\,(x)=\begin{cases} inf\ \{v(x)\ ;\ v\in S_u^F(G)\} & \text{if } S_u^F(G)\neq\phi \\ +\infty & \text{if } S_u^F(G)=\phi, \end{cases}$$

$$R_G^{F,\delta}(u)\,(x)=\inf_{\omega\in\Omega_O} R_G^{F\cap C\omega}(u)\,(x)$$

where we denote by Ω_O the totality of all relatively compact open sets.

For any closed set F, the subset $S_O(F;G)$ of $S(G)$ is defined by

$$S_O(F;G)=\{u\in S(G)\ ;\ R_G^{F,\delta}(u)\,(x)=0\ G\text{-n.e. on } X\}.$$

DEFINITION 2. We say that a closed set F is *G-u-thin at infinity δ* when $u\in S_O(F;G)$.

DEFINITION 3. We say that a closed set F is *G-thin at infinity* δ when $P_{M_0}(G) \subset S_0(F;G)$.

THEOREM 1. *Let G be a symmetric and continuous function-kernel on X satisfying the complete maximum principle and F be a closed set in X. Suppose that every non-empty open set in X is not G-negligible. Then the following three statements are equivalent :*

(1) *F is G-cap. thin at infinity* δ.

(2) (a) *F is G-thin at infinity* δ, *and*

(b) $cap_G^i(F) < +\infty$.

(3) (c) *F is G-1-thin at infinity* δ, *and*

(b) $cap_G^i(\mathrm{F}) < +\infty$.

For the proof of THEOREM 1, we recall here the following well-known results.

PROPOSITION A (*cf.* [8]). *For a continuous function-kernel G on X satisfying the domination principle and for a closed set F in X, the following statements are equivalent :*

(1) *F is G-thin at infinity* δ.

(2) *For every* $\nu \in F_C(G)$, *the potential* $G\nu$ *belongs to* $S_0(F;G)$.

PROPOSITION B (cf. [5]). *Let G be a continuous function-kernel on X and u be a G-superharmonic function on X. We assume that G satisfies the domination principle and that every non-empty open set in X is not G-ngligible. If a closed set F in X is G-u-thin at infinity* δ, *namely* $u \in S_0(F;G)$, *then there exists a positive mesure* μ' *such that*

$$S\mu' \subset F,$$

$$G\mu'(x) = u(x) \ G\text{-}n.e. \ \text{on } F,$$

$$G\mu'(x) \leqq u(x) \ \text{in } X.$$

PROOF OF THEOREM 1. (1)→(2). Suppose that F is *G-cap. thin at* δ. The inequality in (b) is an immediate consequence of the subadditivity of capacity. Therefore it suffices to obtain (a).

Given $\varepsilon > 0$, there exists by (1) an open set $\omega_0 \in \Omega_0$ such that

$$cap_G^i(F \cap C\omega) < \varepsilon \ \text{for every } \omega \in \Omega_0 \ \text{satisfying } \omega \supset \omega_0.$$

For any μ and ν in $F_C(G)$, any compact set K and any $\omega \in \Omega_0$ such that $\omega \supset \omega_0$, we denote by $\mu'_{F \cap C\omega \cap K}$ a balayged measure of μ on the compact set $F \cap C\omega \cap K$. Putting $M = max\ G\mu(x)$ and $N = max\ G\nu(x)$, we have

$$\int R_G^{F \cap C\omega \cap K}(G\mu) d\nu = \int G\mu'_{F \cap C\omega \cap K} d\nu = \int G\nu d\mu'_{F \cap C\omega \cap K}$$
$$\leqq M \int d\mu'_{F \cap C\omega \cap K} \leqq MN \int d\mu'_{F \cap C\omega \cap K}/N \leqq MN \ cap_G^i(F \cap C\omega)$$
$$\leqq MN\varepsilon.$$

Letting K and ω tend to X and ε to 0, we obtain (a).

(2)→(3). Suppose (2). For any $\nu \in F_C(G)$ and $c>0$, we can find by (a) an open set $\omega_0 \in \Omega_0$ such that

$$\int R_G^{F\cap C\omega}(G\nu)\, d\nu < c\varepsilon \textit{ for every } \omega \in \Omega_0 \textit{ satisfying } \omega \supset \omega_0.$$

Put $E=\{x \in X\ ;\ G\nu(x) \geqq c\}$. Given a compact K, we denote by $\gamma_{F\cap C\omega\cap K}$ an equilibrium measure of $F\cap C\omega \cap K$ and by $\nu'_{E\cap F\cap C\omega\cap K}$ a balayaged measure of ν on $E\cap F\cap C\omega\cap K$. Then we have

$$\int R_G^{F\cap C\omega\cap K}(1)d\nu = \int G\gamma_{F\cap C\omega\cap K}d\nu = \int G\nu d\gamma_{F\cap C\omega\cap K}$$
$$= \int_E + \int_{CE}.$$

We shall estimate the last two integrals. First we obtain

$$\int_E G\nu d\gamma_{F\cap C\omega\cap K} = \int_E G\nu'_{E\cap F\cap C\omega\cap K}d\gamma_{F\cap C\omega\cap K}$$
$$= \int d\nu'_{E\cap F\cap C\omega\cap K} \leqq \frac{1}{c}\int G\nu d\nu' \leqq \frac{1}{c}\int R_G^{F\cap C\omega}(G\nu)d\nu$$
$$\leqq \frac{1}{c}c\varepsilon \textit{ for any } \omega \in \Omega_0 \textit{ satisfying } \omega \supset \omega_0.$$

On the other hand, the second integral is estimated as follows.

$$\int_{CE} G\nu d\gamma_{F\cap C\omega\cap K} \leqq c\int_{CE} d\gamma_{F\cap C\omega\cap K} \leqq c\ \mathrm{cap}_G^i(F).$$

Thus we have

$$\int R_G^{F\cap C\omega\cap K}(1)d\nu < \varepsilon + c\ cap_G^i(F).$$

Letting ω and K tend to X and ε and c to 0, we obtain (c).

(3)→(1). Suppose (3). F being *G-1-thin* at δ, there exists, by virtue of PROPOSITION B , an equilibrium measure γ_F of F.
It follows from (c) that

$$\int_K R_G^{F,\delta}(1)d\gamma_F = 0 \quad \textit{for any compact } K.$$

Letting K tend to X, we have

$$\int R_G^{F,\delta}(1)d\gamma_F = 0.$$

Therefore, by the aid of (b), we can find, for a given $\varepsilon>0$, an open set $\omega_0 \in \Omega_0$ such that

$$\int R_G^{F\cap C\omega}(1)\, d\gamma_F \leqq \varepsilon \textit{ for any } \omega \in \Omega_0 \textit{ satisfying } \omega \supset \omega_0.$$

For any compact K and any $\omega \in \Omega_0$, $\gamma_{F\cap C\omega\cap K}$ denotes an equilibrium measure of $F\cap C\omega\cap K$. For any $\omega \in \Omega_0$ such that $\omega \supset \omega_0$ we have

$$\int d\gamma_{F\cap C\omega\cap K} \leqq \int R_G^{F\cap C\omega}(1)d\gamma_{F\cap C\omega\cap K} \leqq \int R_G^{F\cap C\omega}(1)d\gamma_F \leqq \varepsilon.$$

Since K is arbitrary, we can deduce that F is *G-cap. thin at* δ.
This completes the proof.

COROLLARY 1. *Let G be a symmetric and continuous function-kernel satisfying the complete maximum principle and F be a closed set. Suppose that* $cap_G^i(F) < +\infty$. *Then the following three statements are equivalent* :
(1) *F is G-cap. thin at infinity* δ.
(2) *F is G-thin at infinity* δ.
(3) *F is G-1-thin at infinity* δ.

COROLLARY 2. *Let G be a symmetric and continuous function-kernel satisfying the complete maximum principle and F be a closed set. Suppose that F is G-thin at infinity* δ. *Then F is G-cap. thin at infinity* δ *if and only if* $cap_G^i(F) < +\infty$.

Next we characterize the G-thinness of a closed set at infinity δ by the behavior of potentials in the neighborhood of δ.
We begin with the following lemma.

LEMMA. *Let G be a symmetric and continuous function-kernel. Assume that G satisfies the complete maximum principle. Then, for any mesure* μ *in* M_O *and any positive number c, the following inequality holds* :

$$cap_G^i(\{ x \in X \ ; \ G\mu(x) \geqq c\}) \leqq \frac{1}{c} \int d\mu.$$

PROOF. For any compact set K contained in $\{x \in X \ ; \ G\mu(x) \geqq c\}$, we denote by γ_K *an equilibrium measure of* K. Then we have

$$\int d\gamma_K \leqq \frac{1}{c} \int G\mu d\gamma_K = \frac{1}{c} \int G\gamma_K d\mu \leqq \frac{1}{c} \int d\mu.$$

Since K is arbitrary, we obtain the desired inequality.

THEOREM 2. *Let G be a symmetric and continuous function-kernel and F be a closed set. Suppose that G satisfies the complete maximum principle. Then the following statements are equivalent* :
(1) F is G-thin at infinity δ.
(2) *For any measure* ν *in* M_O *and any positive number c, the closed set*

$$F \cap \{ x \in X \ ; \ G\nu(x) \geqq c \}$$

is G-cap. thin at infinity δ.

PROOF. (1)→(2). Assume that F is *G-thin at* δ. By virtue of the above lemma, we have, for any $\nu \in M_O$ and any $c > 0$,

$$cap_G^i (F \cap \{ x \in X \ ; \ G\nu(x) \geqq c \})$$
$$\leqq cap_G^i (\{ x \in X \ ; \ G\nu(x) \geqq c \}) \leqq \frac{1}{c} \int d\nu < +\infty.$$

F being *G-thin at* δ, $F \cap \{ x \in X \ ; \ G\nu(x) \geqq c\}$ is also *G-thin at* δ. Therefore the equivalence betwee (1) and (2) in THEOREM 1 asserts that the clsed set $F \cap \{ x \in X \ ; \ G\nu(x) \geqq c \}$ is *G-cap. thin* *at* δ.

(2)→(1). Suppose that, for any $\nu \in F_C(G)$ and any $c>0$, the set $F \cap \{ x \in X ; G\nu(x) \geqq c \}$ is *G-cap. thin at* δ. For the sake of the simplicity of our description, we put $E = \{ x \in X ; G\nu(x) \geqq c \}$.

It suffices to prove that the equality $\int R_G^{F,\delta}(G\mu)d\nu=0$ holds for every measure μ in $F_C(G)$.

By our assumption (2), the closed set $F \cap E$ is *G-cap. thin at* δ. Therefore THEOREM 1 asserts that $F \cap E$ is *G-thin at* δ. Hence, given $\varepsilon>0$, there exists an open set ω_0 *in* Ω_0 satisfying

$$cap_G^i(F \cap E \cap C\omega) < \varepsilon \text{ for any } \omega \in \Omega_0 \text{ such that } \omega \supset \omega_0.$$

For any $\omega \in \Omega_0$ satisfying $\omega \supset \omega_0$ and compact K, we denote by $\mu'_{F\cap C\omega\cap K}$ a balayaged measure of μ on $F \cap C\omega \cap K$. Then we have

$$\begin{aligned}\int R_G^{F\cap C\omega\cap K}(G\mu)d\nu &= \int G\mu'_{F\cap C\omega\cap K}d\nu = \int G\nu d\mu'_{F\cap C\omega\cap K} \\ &= \int_E G\nu d\mu'_{F\cap C\omega\cap K} + \int_{CE} G\nu d\mu'_{F\cap C\omega\cap K} \\ &\leqq M \int_E d\mu'_{F\cap C\omega\cap K} + c\int d\mu \\ &\leqq MN\ cap_G^i(F\cap E\cap C\omega) + c\int d\mu \\ &\leqq MN\varepsilon + c\int d\mu.\end{aligned}$$

where $M = max\ G\nu(x)$ and $N = max\ G\mu(x)$.

Letting first K and ω tend to X and next ε and c to 0, we can conclude that F *is G-thin at* δ. Thus the proof of theorem is completed.

We close this section by investigating the relations between the *G-1-thinness at infinity* δ and the existence of an equilibrium measure of a closed set.

THEOREM 3. *Let G be a symmetric and continuous function-kernel and F be a closed set. Suppose that every non-empty open set in X is not G-negligible. Then the following statements are equivalent* :

(1) *F is G-1-thin at infinity* δ.

(2) (a) *F is G-thin at infinity* δ, *and*

(b) *there exists an equilibrium mesure* γ_F *of* F.

PROOF. (1)→(2). Obviously (1) implies (a). On the other hand, the implication (1)→(b) is an immediate consequence of PROPOSITION B.

(2)→(1). Supose that (a) and (b) hold. We shall prove that the equality

$$\int R_G^{F,\delta}(1)d\nu=0$$

is valid for every measure ν in $F_C(G)$.

Given $\varepsilon>0$, we can find by (a), an open set ω_0 in Ω_0 satisfying

$$\int R_G^{F\cap C\omega}(G\nu)d\gamma_F<\varepsilon \text{ for every } \omega \in \Omega_0 \text{ such that } \omega \supset \omega_0.$$

For any compact K and any open set $\omega \in \Omega_0$ satisfying $\omega \supset \omega_0$, we denote by $\gamma_{F\cap C\omega\cap K}$ an equilibrium measure of the compact set $F \cap C\omega \cap K$.

Then we have

$$\int R_G^{F\cap C\omega\cap K}(1)d\nu = \int G\gamma_{F\cap C\omega\cap K}d\nu$$
$$= \int G\nu d\gamma_{F\cap C\omega\cap K} \leqq \int R_G^{F\cap C\omega}(G\nu)d\gamma_{F\cap C\omega\cap K}$$
$$\leqq \int R_G^{F\cap C\omega}(G\nu)d\gamma_F < \varepsilon.$$

Letting K and ω tend to X and ε to 0, we have

$$\int R_G^{F,\delta}(1)d\nu = 0.$$

This completes the proof.

3. Characterization of the regularity of function-kernels

DEFINTION 4. A continuous function-kernel G on X is said to be *regular* when the whole space X is *G-thin at infinity* δ, that is, the inclusion relation $P_{M_0}(G) \subset S_0(X;G)$ is valid.

The regularity of kernels plays an important role in the theory of Hunt kernels (cf. for example [1], [2], [7] and [10]).

In this section, we shall characterize the regularity of function-kernels by the behavior of potentials in the neighborhood of infinity δ.

REMARK 2. (cf, [4]and[5]). The author has proved in the previous papers that, for a symmetric and continuous function-kernel G satisfying the domination principle, the following three statements are equivalent :

(1) G is regular.

(2) G has the so-called dominated convergence property :

$$\{\mu_n\}_{n=1}^{\infty} \subset M \text{ and } \mu_n \to \mu \text{ vaguely as } n \to \infty,$$
$$\exists \mu_0 \in M_0 \text{ such that } G\mu_n(x) \leqq G\mu_0(x) \text{ on } X \text{ for all } n.$$
$$\Longrightarrow$$
$$G\mu(x) = \liminf_{n\to\infty} G\mu_n(x) \text{ } G\text{-n.e. on } X.$$

(3) G is strongly balayable, namely, for every G-superharmonic function u dominated by a potential in $P_{M_0}(G)$ and every closed set F, there exists a positive measure μ' such that

$$S\mu' \subset F,$$
$$G\mu'(x) = u(x) \text{ } G\text{-n.e. on } F,$$
$$G\mu'(x) \leqq u(x) \text{ on } X.$$

REMARK 3. Suppose that G is regular. Then, for a closed set F, the following propositions hold (see COROLLARY 2 and THEOREM 3) :

(1) F is *G-cap. thin at infinity* δ if and only if $\mathrm{cap}_G^i(F) < +\infty$.

(2) F *is G-1-thin at infinity* δ if and only if there exists an equilibrium measure of F.

DEFINITION 5. Let u be a non-negative function u on X. We say that *u is equal to 0 G-quasi-everywhere at infinity δ* and write symply $u=0$ *G-q.e. at δ* when, for any $c>0$, the following equality holds :

$$\inf_{\omega \in \Omega_0} cap_G^i \{ x \in C\omega \ ; \ u(x) \geqq c \} = 0.$$

THEOREM 4. *For a symmetric and continuous function-kernel G satisfying the complete maximum principle, the following statements are equivalent :*

(1) *G is regular.*
(2) *For every $\mu \in M_0$, $G\mu=0$ G-q.e. at infinity δ.*

PROOF. By our definition, G is *regular* if and only if X is *G-thin at infinity δ*. On the other hand, $G\mu=0$ *G-n.e. at δ* if and only if, for every $c>0$, the set $\{x \in X \ ; \ G\mu(x) \geqq c\}$ is *G-cap. thin at δ*. Put $F=X$ in THEOREM 2 and our theorem follows immediately.

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(Received Feb.20.1994)