愛知工業大学研究報告
第 40 号 B 平成 17 年

Distribution of Direct Measurement Result of Measurement Point of Form Error

形状誤差における測定点の直接測定結果の分布

Xiaohua NI[†], Yoshihisa UCHIDA[††]
倪驍驊[†]　内田敬久[††]

Abstract The uncertainty of measurement result of form error is influenced by the uncertainties of measurement data that is used during the processing of the calculation. According to the properties of the measurement of form error, by means of Maximum entropy method and the theory of probability and statistics, the probability distributions of reading value and direct measurement result of measurement points are deduced. It is concluded that it is reasonable to regard the uncertainty distributions of reading value as uniform distribution, and the uncertainty distributions of direct measurement result of measurement points as normal distribution.

1. Introduction

The uncertainty of measurement result of form error is influenced by the data at measurement point that is used during the processing of the calculation of form error. During the measurement of form error, the data is called reading value that is read by operator or collected by computer at the measurement point. Generally it could not be used to estimate the form error, especially when the measurement is un-direction. Usually the reading value should be deal with to get rid of those errors that are caused by the movement of instrument, the estimation reading of operator, measurement circumstance, the impaction and destination of parts and so on. The data is called direct measurement result that could be directly used to estimate the form error and any transform is not necessary. Sometimes the direct measurement result of each point is calculated by cumulation or coordinates transform with reading values. Certainly the direct measurement result is equal to the reading value for some measurement method. In the simplified calculation, sometimes the reading value of instrument is regarded as direct measurement result, and do not eliminate any errors.

Since the direct measurement result is directly used to estimate the form error to calculate the final measurement result, so the uncertainty of it has great contribution to the uncertainty of final measurement result. During the measurement of form error, the total uncertainty of reading value of measurement point is also influenced by many factors. It is gained by the combination with the uncertainties of these factors. The decision of the coverage factor k of B type uncertainty evaluation is influenced by the probability distribution of these factors[1)], also the combined uncertainty of reading value must be decided, because it will influence the decision of direct measurement result of point and

† Department of Mechanical Engineering, Yancheng Institute of Technology (Yancheng, China) and Visiting Scholar of Aichi Institute of Technology (Toyota, Japan)

†† Department of Mechanical Engineering, Aichi Institute of Technology (Toyota, Japan)

the estimation of the uncertainty of final measurement result. Although the probability distribution of measurement result may be gained by statistic of repeat experiments and assumption test, such as Histogram method and Probability paper method, it is useful only to the limited distributions and do not have explicit judgment boundary.

During the measurement of form error, at the same measurement point, measurement times is usually no more than one time, the other influence factors is only known as a value between an interval. So the probability distribution of direct measurement result of form error could not be estimated with popular probability estimation method. In this paper, Maximum entropy method is used to estimate the probability distribution of direct measurement result of form error.

2. Probability distribution estimation by means of Maximum entropy method

A discrete information source may be expresses as follows:

$$x:\begin{bmatrix} x_1 & x_2 & \Lambda & x_n \\ p_1 & p_2 & \Lambda & p_3 \end{bmatrix} \qquad [1]$$

That's to say, if discrete variable x is given the value x_i, the probability p_i, $i = 1, 2, \cdots, n$. Here,

$$p(x = x_i \ \mathrm{I} \ x = x_j) = 0, i \neq j \qquad [2]$$

$$\sum_{i=1}^{n} p_i = 1 \qquad [3]$$

Then the entropy is defined

$$H(x) = H(p_1, p_2, \Lambda, p_n) = -k\sum_{i=1}^{n} p_i \log p_i \qquad [4]$$

The constant k is decided by the unit, usually, it is taken as k=1. The different base of logarithm is given, the entropy will have different unit. From the convenience of calculation, it is given the natural logarithm. Thus the unit of $H(x)$ is "Nat" and the quantum H is called information entropy. It is used to describe the uncertainty of information source. To continuous information source, the distribution of x is described with probability density $p(x)$. So in the case of continuous distribution, the entropy is expressed as

$$H(x) = H(p(x)) = -\int_{-\infty}^{+\infty} p(x)\ln p(x)dx = E[\ln p(x)] \qquad [5]$$

In another words, the means of the logarithm of distribution density $p(x)$ is entropy.

The maximum of entropy may be used to estimate the probability distribution, the actual calculation method is introduced in references (2) - (4). With this method, no other subjective assumption is needed but measurement data error that contains the all information and constrained condition of samples. That is to say, in the most of uncertainty, the actual probability distribution and its parameters are estimated with maximum entropy rule[4)].

3. The distribution of reading value of measurement point

During the measurement, the uncertainty interval of reading value may be taken from the handbook of instrument, suppose the estimation of reading value x is μ, so the true value of it is among an interval, suppose the interval is $[a, b]$. Generally the interval is symmetrical where μ is center point. Now let the interval to be changed to make the center point value to zero, thus the shape of the distribution do not be influenced. Also let $L = (b-a) / 2$, so the interval becomes $[-L, L]$.

Approximately the probability density $p(x)$ fulfils

$$\int_{-L}^{L} p(x) = 1 \qquad [6]$$

From formula [5], there will be the entropy function

$$H(x) = H(p(x)) = -\int_{-L}^{L} p(x)\ln p(x)dx \qquad [7]$$

To get extremum, Lagrange method of multipliers is used, suppose

$$D = -\int_{-L}^{L} p(x)\ln p(x)dx + (\lambda_0 + 1)[\int_{-L}^{L} p(x)dx - 1] \quad [8]$$

Let,

$$\frac{\partial D}{\partial p} = 0 \quad [9]$$

Then

$$-\int_{-L}^{L}[\ln p(x) + 1]dx - (\lambda_0 + 1)\int_{-L}^{L} dx = 0 \quad [10]$$

So

$$-\ln p(x) + \lambda_0 + 1 = 0 \quad [11]$$

Thus

$$p(x) = \exp(-1 - \lambda_0) \quad [12]$$

From equation [6],

$$\exp(-1 - \lambda_0) = \frac{1}{2L} \quad [13]$$

So we get

$$p(x) = \frac{1}{2L}, x \in [-L, L] \quad [14]$$

This is uniform distribution. That is to say, in the interval [-*L*, *L*], it is uniform distribution that have the maximum entropy, so the distribution function of reading value may be deduced that

$$f(x) = \frac{1}{b - a}, x \in [a, b] \quad [15]$$

4. The distribution of direct measurement result of measurement point

During the calculation of uncertainty of direct measurement result of measurement point, the uncertainty of reading value is only a factor of the uncertainty of measurement point data. It is also influenced by movement of instrument, measurement circumstance and the impaction and destination of parts and so on. Although the numerical value of measurement point data is equal to reading value, the uncertainty is larger.

It is known that the uncertainty of measurement point data is combined with the uncertainties of many factors, the distribution of these factors may obeys uniform distribution, normal distribution or other non-normal distribution, since the measurement times is few. So the distribution of combined result could not be estimated with popular statistic method. Therefore, the Maximum entropy method is also used.

Suppose the uncertainty of measurement point data is known, so it is considered that the variance is known, let it be σ^2, the estimation of measurement point data is μ, not let it be zero, μ become variable *x*, then the variance σ^2 is second origin moment, suppose the distributing bound of *x* is $x \in [-\infty, +\infty]$. Then

$$\int_{-\infty}^{+\infty} p(x)dx = 1 \quad [16]$$

$$\int_{-\infty}^{+\infty} xp(x)dx = 0 \quad [17]$$

$$\int_{-\infty}^{+\infty} x^2 p(x)dx = \sigma^2 \quad [18]$$

From reference (4), it is known that normal distribution make the entropy to maximum. So

$$p(x) = \frac{1}{\sigma^2\sqrt{2\pi}}\exp(-\frac{x^2}{2\sigma^2}), x \in [-\infty, +\infty] \quad [19]$$

Then the distribution function of total uncertainty of measurement point data that makes the entropy to get a maximum should be

$$f(x)=\frac{1}{\sigma^2\sqrt{2\pi}}\exp[-\frac{(x-\mu)^2}{2\sigma^2}] \quad , x\in [-\infty, +\infty] \qquad [20]$$

It is also normal distribution.

During the measurement of form error, whether the measurement is direct or not, the direct measurement result at each measurement points may be expressed as linear function of measurement point data, because the measurement point data is normal probability variable and independent from each other; from the theory of probability, the linear function of normal random variable is also normal random variable. Therefore, it is deduced that the direct measurement result of measurement point obeys to normal distribution.

5. Summary

The uncertainty of measurement result of form error is influenced by the data at measurement point that is used during the processing of the calculation of form error. According to the properties of the measurement of form error, the modern probability distribution estimation method ---- Maximum entropy method is used to deduce the probability distribution of reading value and direct measurement result. It is deduced that the distribution of reading value obeys uniform distribution and the distribution of direct measurement result obeys normal distribution. The conclusion will give convenience for estimation of total uncertainty of the measurement result of form error.

References

1) BIPM-IEC-IFCC-ISO-IUPAP-OIML. Guide to the Expression of Uncertainty in Measurement. ISO, pp.1-6, 1995
2) Ling Honghua, “Instruction of the methods of non-normal error statistics processing and estimation” Special Papers for Measurement Uncertainty, Statute Department of National Metrology Bureaus, pp.67-74, 1985
3) Wang Zhongyu, Xia Xintao, Zhu Jianmin, “The Un-Statistic Theory of Measurement Uncertainty”, National Defense Publishing Company, pp.100-119, 2000
4) Zhou Zhaojin, “A principle of MEM to estimate the measurement uncertainty”, Measurement Technique, No.4, pp.1-3, 1989

（受理　平成 17 年 3 月 17 日）